STUDENT WORKBOOK

College Algebra

WITH REVIEW

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Developed by Jenifer Bohart
Scottsdale Community College

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Jenifer Bohart, Scottsdale Community College

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Unit 1: Introduction to Functions

Section 1.1: Relations and Functions

Definitions
A **RELATION** is any set of ordered pairs.   A **FUNCTION** is a relation in which **every** input value is paired with **exactly one** output value

Table of Values

One way to represent the relationship between the input and output variables in a relation or function is by means of a table of values.

Example 1: Which of the following tables represent functions?

Input	Output
1	5
2	5
3	5
4	5

Yes No

Input	Output
1	8
2	–9
3	7
3	12

Yes No

Input	Output
2	4
1	–5
4	10
–3	–87

Yes No

Ordered Pairs

A relations and functions can also be represented as a set of points or ordered pairs.

Example 2: Which of the following sets of ordered pairs represent functions?

A = {(0, –2), (1,4), (–3,3), (5,0)}

B = {(–4,0), (2, –3), (2, –5)}

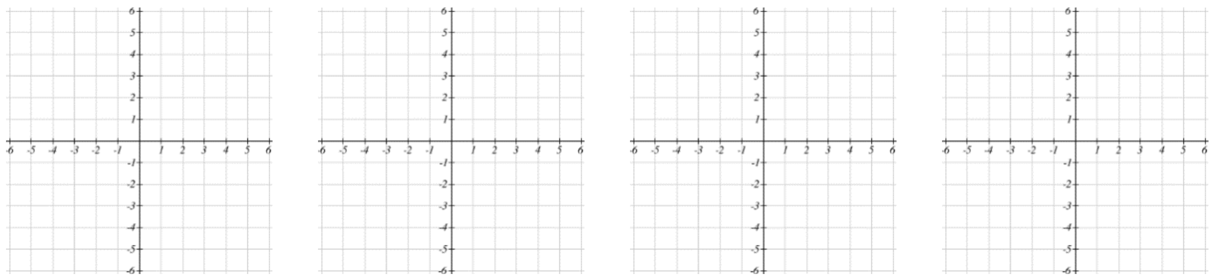
C = {(–5,1), (2,1), (–3,1), (0,1)}

D = {(3, –4), (3, –2), (0, 1), (2, –1)}

E = {(1,3)}

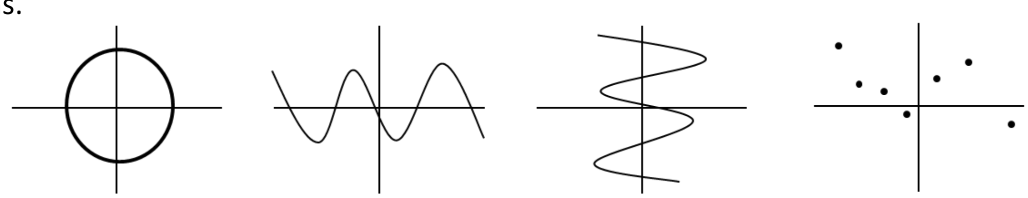
Example 3: On the graphs below, plot the points for A, B, C, and D from Example 2, then circle the "problem points"

A B C D



The Vertical Line Test
• If all vertical lines intersect the graph of a relation at no more than one point, the relation *is* also a function. One and only one output value exists for each input value. • If any vertical line intersects the graph of a relation at more than one point, the relation "fails" the test and is NOT a function. More than one output value exists for some (or all) input value(s).

Example 4: Use the Vertical Line Test to determine which of the following graphs are functions.



Behavior of Graphs

Increasing	Decreasing	Constant

Dependent and Independent Variables
In general, we say that the output **depends** on the input. Output variable = **Dependent Variable** Input Variable = **Independent Variable** If the relation is a function, then we say that the output **is a function of** the input.

Section 1.2: Function Notation: *f* (input) = output

If a relation is a function, we say that the *output is a function of the input.*

Function Notation: *f* (input) = output

Example: If y is a function of x, then we can write $f(x) = y$.

Example 1: The function V(m) represents value of an investment (in thousands of dollars) after m months. Explain the meaning of V(36) = 17.4.

Ordered Pairs

Example 2:

Ordered Pair (input, output)	Function Notation *f* (input) = output
(2, 3)	*f* (2) = 3
(–4, 6)	*f* (_____) = ______
(______ , ______)	*f* (5) = –1

Example 3: Consider the function: $f = \{(2, -4), (5, 7), (8, 0), (11, 23)\}$

f(5) = __________ *f*(______) = 0

Table of Values

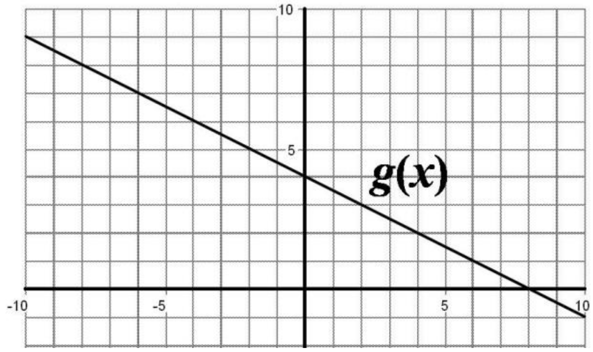
Example 4: The function $B(t)$ is defined by the table below.

T	1	3	12	18	22	31
***B*(*t*)**	70	64	50	39	25	18

$B(12)$ = _________________________ $B(t)$ = 18 when t = ______________________

Graphs

Example 5: Consider the graph $g(x)$ of shown below



$g(2)$ = __________

Ordered pair:____________

$g($______$) = 2$

Ordered pair:____________

$g(0)$ = __________

Ordered pair:____________

$g($______$) = 1$

Ordered pair:____________

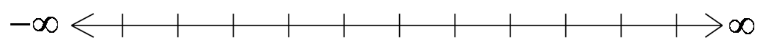
Section 1.3: Inequalities and Interval Notation

Graph the given intervals and then express using interval notation:

x > 2

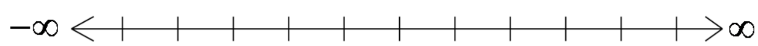
Interval Notation

x ≤ 2



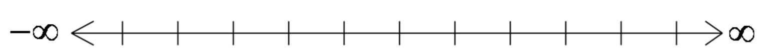
Interval Notation

-3 ≤ x < 4



Interval Notation

x ≤ -3 OR x > 4



Interval Notation

Section 1.4: Domain and Range

DEFINITIONS
The **DOMAIN** of a function is the set of all possible values for the **input** variable.   The **RANGE** of a function is the set of all possible values for the **output** variable.

DOMAIN AND RANGE

Example 1: Consider the function below

x	–2	0	2	4	6
$k(x)$	3	–7	11	3	8

Input values ____

Domain: {_______________________________}

Output values:_______________________________________

Range: {_______________________________}

Example 2: Consider the function: $B = \{(2, -4),\ (5, 7),\ (8, 0),\ (11, 23)\}$

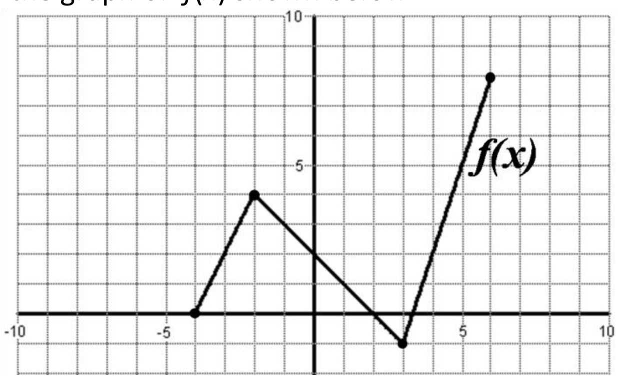
Input values ____

Domain: {_______________________________}

Output values:_______________________________________

Range: {_______________________________}

Example 3: Consider the graph of $f(x)$ shown below

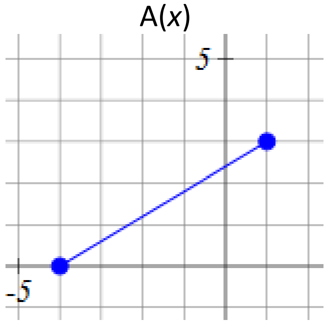


Domain: ______________ $\leq$ ***x*** $\leq$ _______________

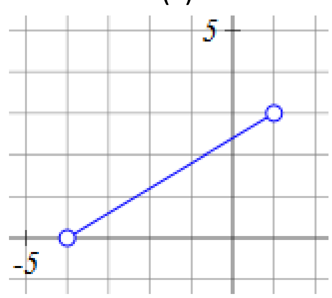
Range: ______________ $\leq$ ***f*(*x*)** $\leq$ _______________

Example 4: Determine the Domain and Range of each of the following graphs:

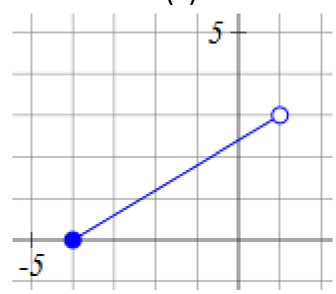
A(x)



B(x)



C(x)

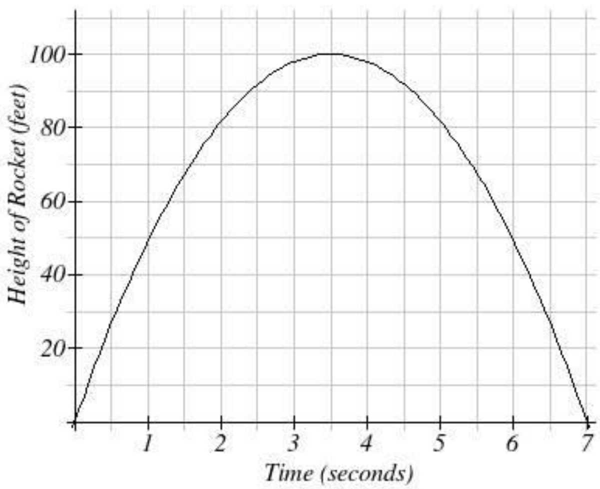


Domain Domain Domain

Range Range Range

Section 1.5: Applications

Example 1: Consider the graph of the function H(*t*) shown below.



Input Variable: __________________

Units of Input Variable: ___________

Output Variable: ________________

Units of Output Variable: _________

a. Interpret the meaning of the statement H(5)=82.

b. Determine H(7). Write it as an ordered pair and interpret its meaning in a complete sentence.

c. Determine *t* when H(*t*) = 50. Write it as an ordered pair and interpret its meaning in a complete sentence.

d. Determine the maximum height of the rocket.

e. Determine the practical domain for H(*t*).

f. Determine the practical range for H(*t*).

Section 1.6: Formulas in Function Notation

Example 1: Let $f(x) = x^2 - 2x + 11$

a. Determine $f(-3)$

b. Determine $f(0)$

Example 2: Let $h(x) = 2x - 5$

a. Determine $h(4)$

b. For what value of x is $h(x) = 17$?

Example 3: Let $g(x) = 71$

a. Determine $g(5)$.

b. Determine $g(-40)$.

Example 4: Let $f(x) = 2x - 5$

a. Determine $f(2)$.

b. Determine $f(-1)$.

c. Determine f(x+1)

d. Determine $f(-x)$

Section 1.7: Applications

Example: Grace is selling snow cones at a local carnival. Her profit, in dollars, from selling x snow cones is given by the function $P(x) = 2.5x - 30$.

a. Write a complete sentence to explain the meaning of $P(30) = 45$ in words.

b. Determine $P(10)$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

c. Determine $P(0)$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

d. Determine x when $P(x) = 100$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

e. Determine x when $P(x) = 0$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

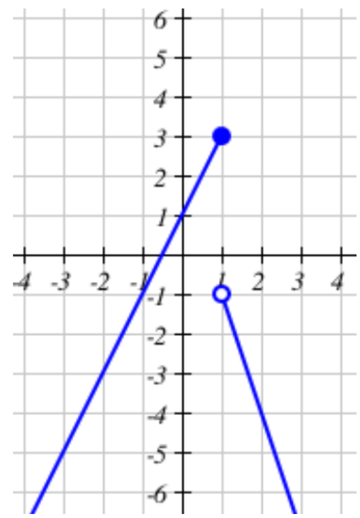
Ordered Pair: ____________

Section 1.8: Piecewise Functions

Piecewise Functions: $f(x) = \begin{cases} \text{Formula 1} & \text{if} \quad \text{Domain for Formula 1} \\ \text{Formula 2} & \text{if} \quad \text{Domain for Formula 2} \\ \text{Formula 3} & \text{if} \quad \text{Domain for Formula 3} \end{cases}$

Example 1: Determine each function value.

$$f(x) = \begin{cases} 2x + 1 & \text{if} \quad x \leq 1 \\ -3x + 2 & \text{if} \quad x > 1 \end{cases}$$



f(-3) =

f(2) =

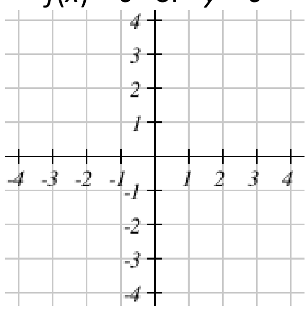
f(1) =

Section 1.9: Toolkit (Parent) Functions

Example 1: Sketch the graph and fill in a table of values for each of the functions below.

Constant Function

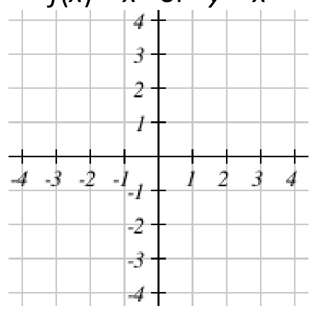
$f(x) = c$ or $y = c$



x	y

Identity Function

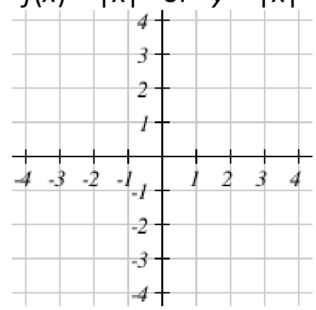
$f(x) = x$ or $y = x$



x	y

Absolute Value Function

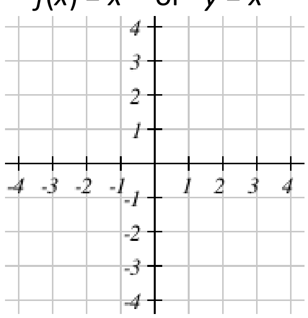
$f(x) = |x|$ or $y = |x|$



x	y

Quadratic Function

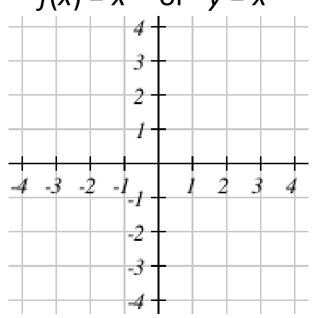
$f(x) = x^2$ or $y = x^2$



x	y

Cubic Function

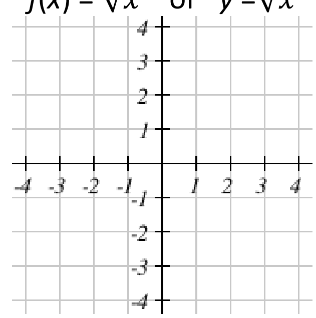
$f(x) = x^3$ or $y = x^3$



x	y

Square Root Function

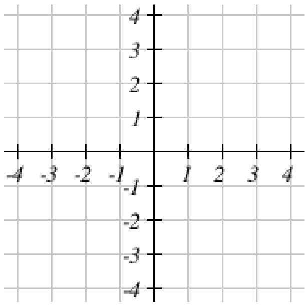
$f(x) = \sqrt{x}$ or $y = \sqrt{x}$



x	y

Cube Root Function

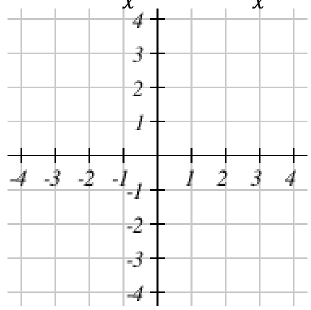
$f(x) = \sqrt[3]{x}$ or $y = \sqrt[3]{x}$



x	y

Reciprocal Function

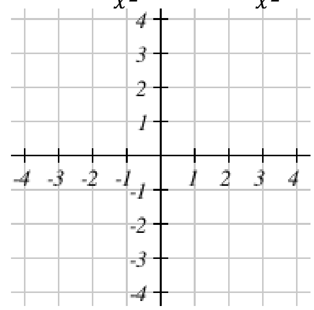
$f(x) = \frac{1}{x}$ or $y = \frac{1}{x}$



x	y

Reciprocal Squared Function

$f(x) = \frac{1}{x^2}$ or $y = \frac{1}{x^2}$

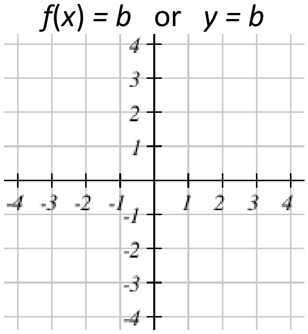


x	y

Example 2: Determine the domain and range of each of the functions below.

Constant Function

$f(x) = b$ or $y = b$

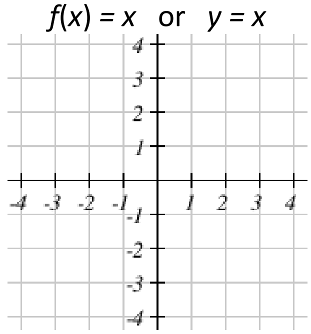


Domain

Range

Identity Function

$f(x) = x$ or $y = x$

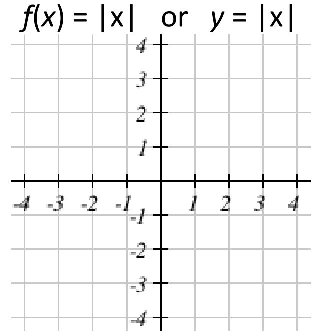


Domain

Range

Absolute Value Function

$f(x) = |x|$ or $y = |x|$

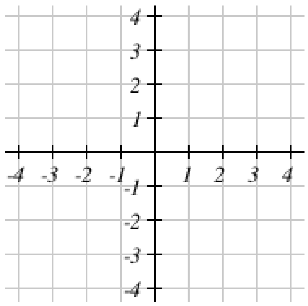


Domain

Range

Quadratic Function

$f(x) = x^2$ or $y = x^2$

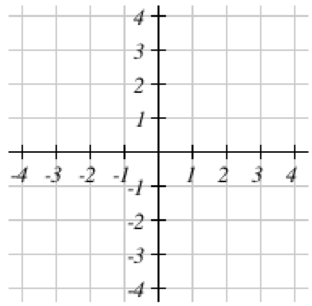


Domain

Range

Cubic Function

$f(x) = x^3$ or $y = x^3$

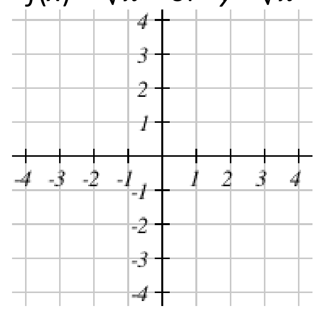


Domain

Range

Square Root Function

$f(x) = \sqrt{x}$ or $y = \sqrt{x}$

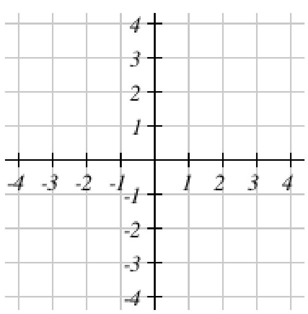


Domain

Range

Cube Root Function

$f(x) = \sqrt[3]{x}$ or $y = \sqrt[3]{x}$

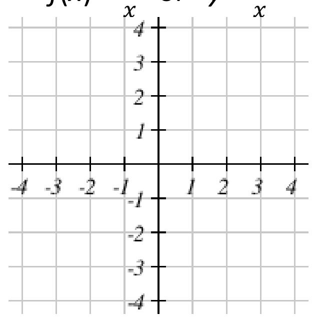


Domain

Range

Reciprocal Function

$f(x) = \frac{1}{x}$ or $y = \frac{1}{x}$

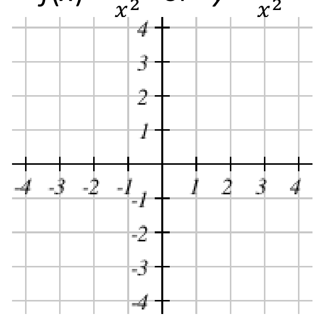


Domain

Range

Reciprocal Squared Function

$f(x) = \frac{1}{x^2}$ or $y = \frac{1}{x^2}$



Domain

Range

Section 1.10: Using Your Graphing Calculator

Example 1: Linear Equation: $A = 8 - 2n$

a) Use the TABLE feature of your graphing calculator to complete the table below.

n	-5	0	2	7	9	12
A						

b) Use your graphing calculator to sketch the graph of $A = 8 - 2n$. Use the indicated viewing windows. Draw what you see on your calculator screen.

Standard Viewing Window
xmin= -10, xmax= 10,
ymin= -10, ymax= 10

xmin= 0, xmax= 5,
ymin= -2, ymax= 8

Example 2: Exponential Equation: $P = 28(1.17)^t$

a) Use the TABLE feature of your graphing calculator to complete the table below. Round to the nearest tenth as needed.

t	-15	-5	0	5	10	15
P						

b) Use your graphing calculator to sketch the graph of $P = 28(1.17)^t$. Use the indicated viewing windows. Draw what you see on your calculator screen.

Standard Viewing Window
xmin= -10, xmax=10,
ymin= -10, ymax=10

xmin= -15, xmax= 15,
ymin= 0, ymax= 300

Unit 2: Transformations of Functions

Section 2.1: An Overview of Transformations

Summary of Transformations

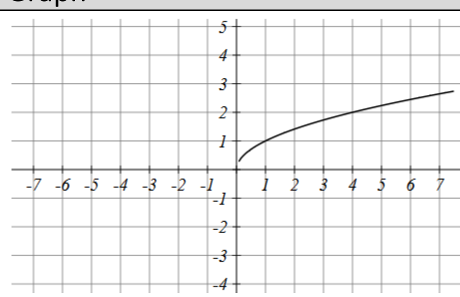
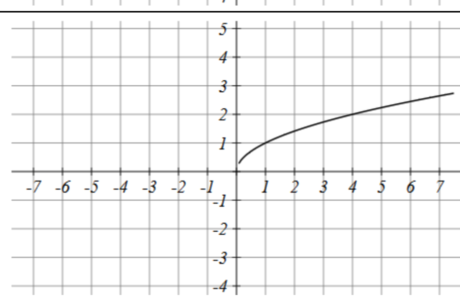
Assume c and d are positive constants.

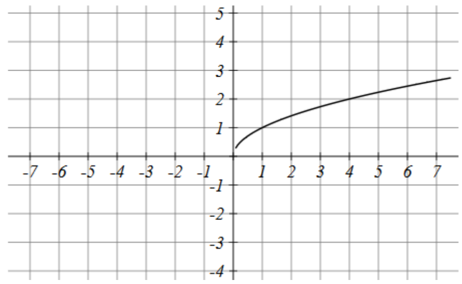
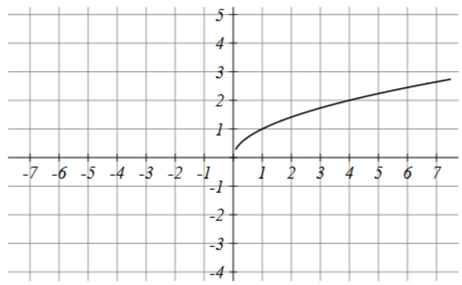
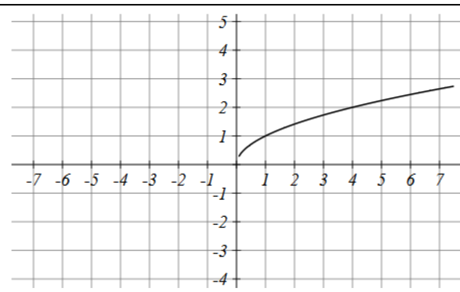
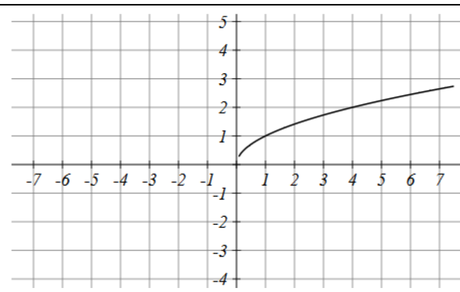
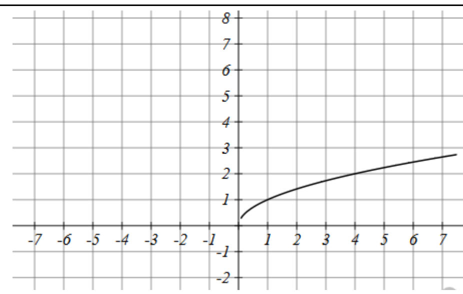
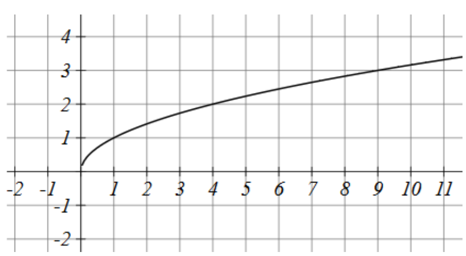
Translations: *f*(x) + d shifts the graph of *f*(x) UP d units
f(x) – d shifts the graph of *f*(x) DOWN d units
f(x + c) shifts the graph of *f*(x) LEFT c units
f(x – c) shifts the graph of *f*(x) RIGHT c units

Reflections: –*f*(x) reflects graph of *f*(x) about the x-axis
f(–x) reflects graph of *f*(x) about the y-axis

Vertical Stretches and Compressions: *af*(x) multiplies the output by a factor of *a*
If *a* > 1, this results in a vertical stretch
If 0 < *a* < 1, this results in a vertical compression

Example 1: For each of the following transformations, draw a sketch of the graph and write down a description of the resulting transformation. The original function is $f(x) = \sqrt{x}$

Transformed Function	Graph	Description and Notes
$f(x) = \sqrt{x} + 2$		
$f(x) = \sqrt{x} - 2$		

$f(x) = \sqrt{x+2}$		
$f(x) = \sqrt{x-2}$		
$f(x) = -\sqrt{x}$		
$f(x) = \sqrt{-x}$		
$f(x) = 3\sqrt{x}$		
$f(x) = 0.5\sqrt{x}$		

Section 2.2: Translations

Assume c and d are positive constants.

Translations: *f*(x) + d shifts the graph of *f*(x) UP d units
f(x) – d shifts the graph of *f*(x) DOWN d units
f(x + c) shifts the graph of *f*(x) LEFT c units
f(x – c) shifts the graph of *f*(x) RIGHT c units

Example 1: Describe the type of translation that would occur to the basic function *f*(x)

f(x) + 4

f(x) – 4

f(x + 4)

f(x – 4)

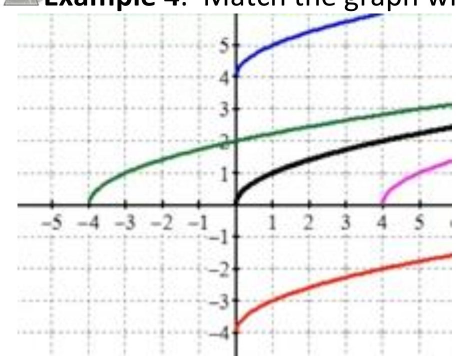
Example 2: Complete the table.

Toolkit Function	Shifting Up: *f*(x) + 3	Shifting Down: *f*(x) – 3
$f(x) = \lvert x \rvert$		
$f(x) = x^2$		
$f(x) = x^3$		
$f(x) = \frac{1}{x}$		
$f(x) = \sqrt{x}$		

Example 3: Complete the table.

Toolkit Function	Shifting Left: $f(x + 3)$	Shifting Right: $f(x - 3)$
$f(x) = \|x\|$		
$f(x) = x^2$		
$f(x) = x^3$		
$f(x) = \frac{1}{x}$		
$f(x) = \sqrt{x}$		

Example 4: Match the graph with the correct function.



a) $g(x) = \sqrt{x+4}$

b) $g(x) = \sqrt{x} + 4$

c) $g(x) = \sqrt{x-4}$

d) $g(x) = \sqrt{x} - 4$

Section 2.3: Reflections

Reflections

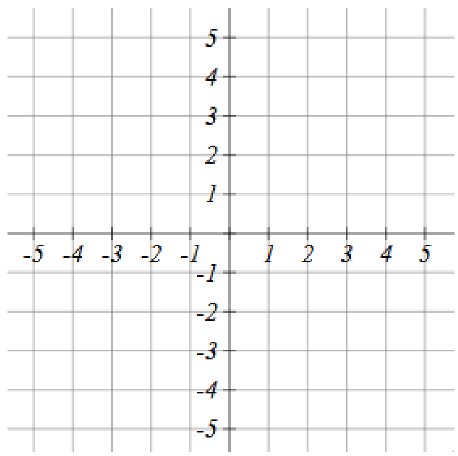
- –*f*(x) reflects graph of *f*(x) about the x-axis
- *f*(–x) reflects graph of *f*(x) about the y-axis

**Example 1**: Reflections of a given point

Give the coordinates of the point (-5, 4) if reflected across the x-axis: _______________

Give the coordinates of the point (-5, 4) if reflected across the y-axis: _______________

Give the coordinates of the point (-5, 4) if reflected across the origin: _______________



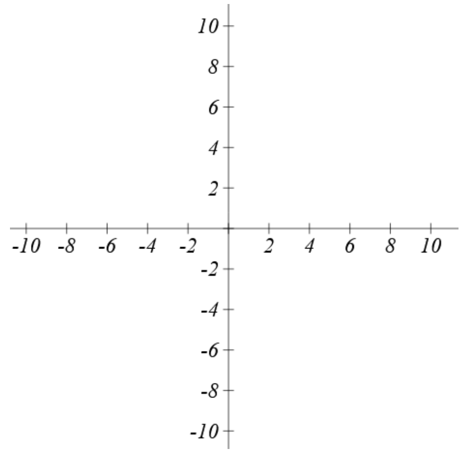
NOTES: To reflect the point (x,y):

- About the x-axis, replace y with -y (change the sign of the y-coordinate)
- About the y-axis, replace x with -x (change the sign of the x-coordinate)
- About the origin, replace x with -x and y with -y (change the sign of the x-coordinate and the y-coordinate)

Example 2: State the domain and then graph the following functions

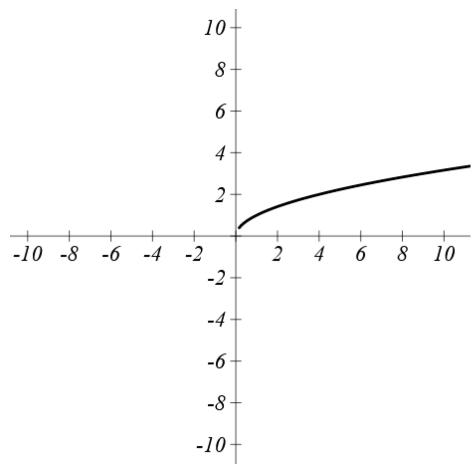
$f(x) = \sqrt{x}$

x	y



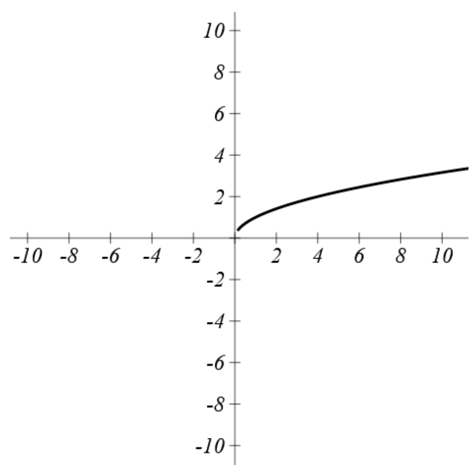
$f(x) = -\sqrt{x}$

x	y



$f(x) = \sqrt{-x}$

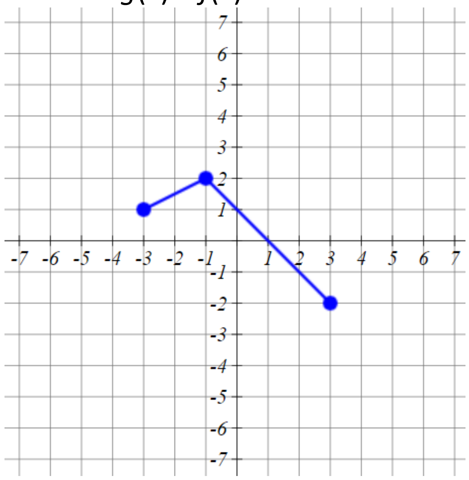
x	y



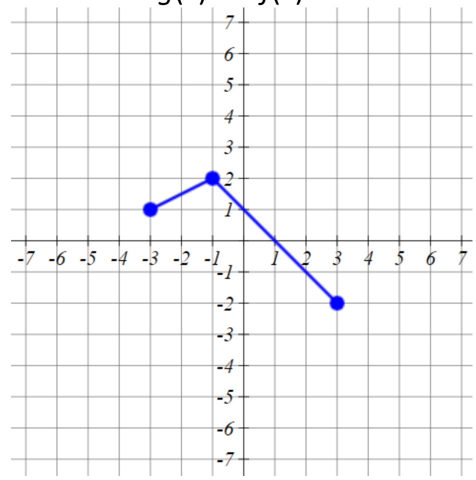
Section 2.4: Function Transformations Given a Graph

Example 1: The graph of *f*(x) is shown. Draw the graph of *g*(x) for each of the following.

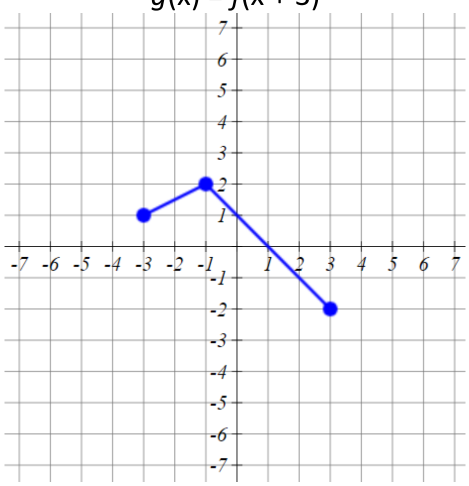
g(x) = *f*(x) + 3



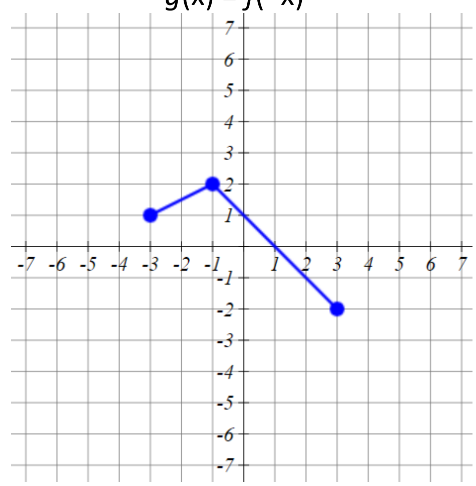
g(x) = – *f*(x)



g(x) = *f*(x + 3)



g(x) = *f*(–x)



Section 2.5: Even and Odd Functions

Even Functions:

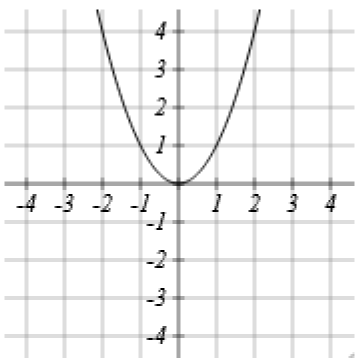
- $f(x) = f(-x)$
- The graph is symmetric across the y-axis

Odd Functions:

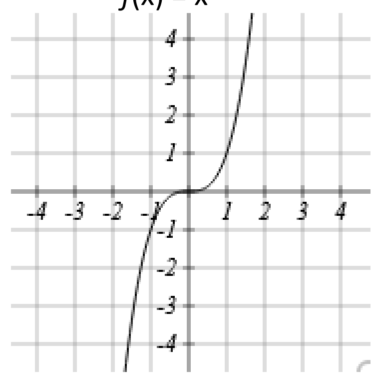
- $-f(x) = f(-x)$
- The graph has rotational symmetry about the origin

Example 1: Determine if the function is even, odd, or neither.

$f(x) = x^2$

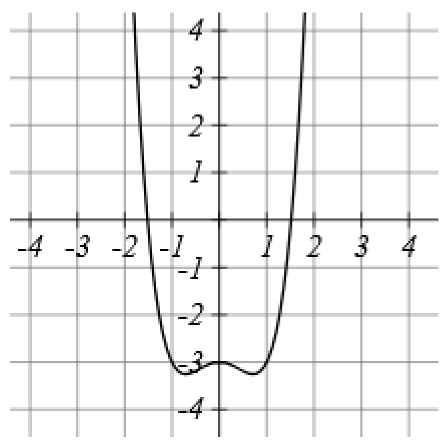


$f(x) = x^3$



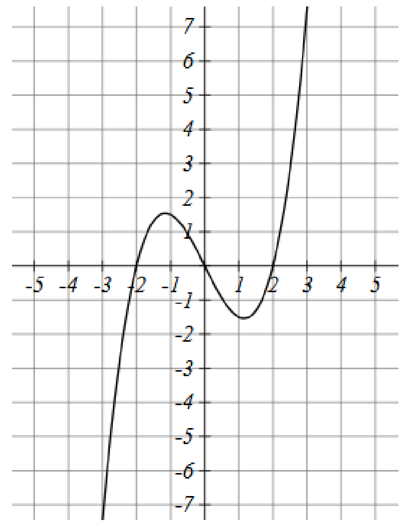
Example 2: Determine if the function is even, odd, or neither.

$f(x) = x^4 - x^2 - 3$



Example 3: Determine if the function is even, odd, or neither

$f(x) = \frac{1}{2}x^3 - 2x$



Section 2.6: Vertical Stretches and Compressions

Vertical Stretches and Compressions

- $af(x)$ multiplies the output by a factor of a
  - If $a > 1$, this results in a vertical stretch
  - If $0 < a < 1$, this results in a vertical compression

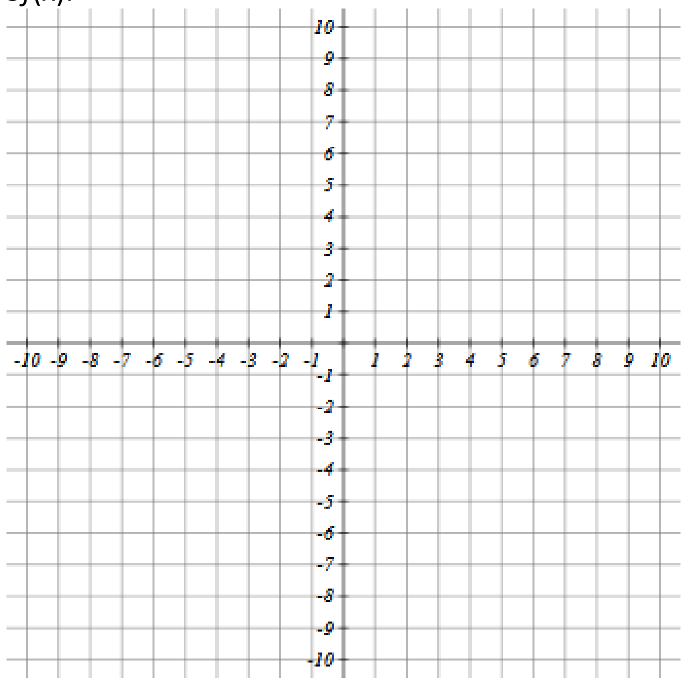
Example 1: If (-3,-2) is a point on the graph of $f(x)$, identify which of the following ordered pairs must be on the graph of $g(x) = 3f(x)$.

(-9,-2)

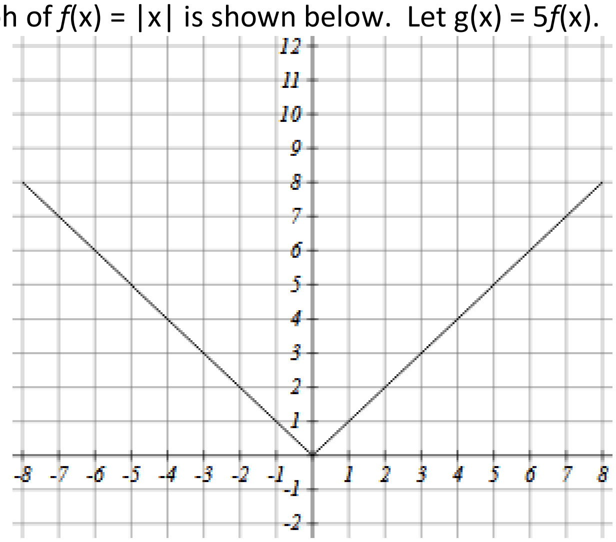
(-9,-6)

(-3,-6)

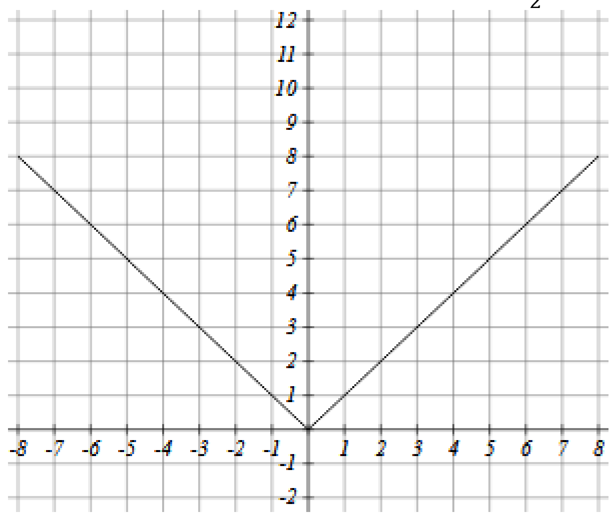
(-3,-9)



Example 2: The graph of $f(x) = |x|$ is shown below. Let $g(x) = 5f(x)$. Draw the graph of $g(x)$.



Example 3: The graph of $f(x) = |x|$ is shown below. Let $g(x) = \frac{1}{2}f(x)$. Draw the graph of g(x).



Section 2.7: Combining Transformations

Assume c and d are positive constants.

Translations

- *f*(x) + d shifts the graph of *f*(x) UP d units
- *f*(x) – d shifts the graph of *f*(x) DOWN d units
- *f*(x + c) shifts the graph of *f*(x) LEFT c units
- *f*(x – c) shifts the graph of *f*(x) RIGHT c units

Reflections

- –*f*(x) reflects graph of *f*(x) about the x-axis
- *f*(–x) reflects graph of *f*(x) about the y-axis

Vertical Stretches and Compressions

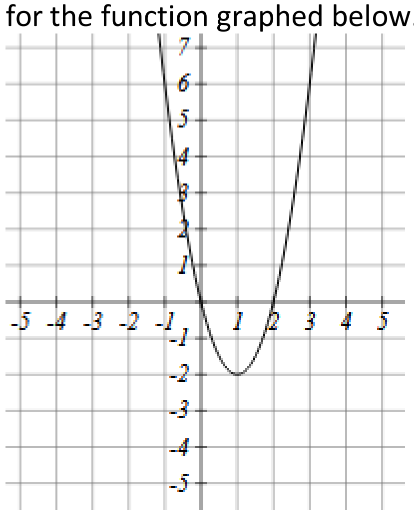
- *af*(x) multiplies the output by a factor of *a*
  - If $a > 1$, this results in a vertical stretch
  - If $0 < a < 1$, this results in a vertical compression

Example 1: Describe a function g(x) in terms of *f*(x) if the graph of g is determined by:

- Vertically stretching f by a factor of 3.
- Shifting the graph to the right 2 units
- Shifting the graph upward 6 units

The transformation of f(x) is ____

Example 2: The graph below is a transformation of the function $y = x^2$. Write an equation for the function graphed below.



Unit 3: Function Operations, Composition, and Inverse

Section 3.1: Properties of Exponents

Given any real numbers *a, b, c, m,* and *n*			
$n^1 =$ _______	$1^n =$ _______	$n^0 =$ _______ $n \neq 0$	$0^n =$ _______ $n \neq 0$

$3^4 =$ _______

$3^3 =$ _______

$3^2 =$ _______

$3^1 =$ _______

$3^0 =$ _______

$3^{-1} =$ _______

$3^{-2} =$ _______

$3^{-3} =$ _______

$3^{-4} =$ _______

Multiplication Properties of Exponents	
$a^m \cdot a^n = a^{m+n}$	$(a^m)^n = a^{mn}$
Why?	Why?

Example 1: Evaluate and simplify the following expressions.
Assume $x \neq 0$, $x \neq -1/2$, $a \neq 0$, $b \neq 0$, and $c \neq 0$.

$5x^0$ $(2x + 1)^0$ $a^0 + b^0 + c^0$

The Multiplication Property: $a^m \cdot a^n = a^{m+n}$

Example 2: Simplify the following expressions

n^3n^9 $b^5 \cdot b^4 \cdot b$ $5x^2y^5(7xy^9)$

Raising a Power to a Power: $(a^m)^n = a^{mn}$

Example 3: Simplify the following expressions

$(x^3)^9$ $5b^2(b^5)^8$

Raising a Product to a Power: $(ab)^n = a^n b^n$

Example 4: Simplify the following expressions

$(5x)^2$ $(x^3y^2)^9$ $(-8ab^5)^2$

$5(-2w^7)^3$ $5n^4(-3n^3)^2$

The Division Property: $\frac{a^m}{a^n} = a^{m-n}$ for $a \neq 0$

Example 5: Simplify the following expressions. Variables represent nonzero quantities.

$\frac{x^{50}}{x^4} =$ $\frac{4a^{10}b^5}{6ab^2} =$

Raising a Quotient to a Power: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ for $b \neq 0$

Example 6: Simplify the following expressions. Variables represent nonzero quantities.

$\left(\frac{5}{7}\right)^2 =$ $\left(\frac{x^5}{y^3}\right)^4 =$ $\left(\frac{-4t^{10}}{u^6}\right)^2 =$

Section 3.2: Operations with Polynomial Expressions

Definitions
Polynomial: An algebraic expression composed of the sum of terms containing a single variable raised to a non-negative integer exponent.
Monomial: A polynomial consisting of **one** term
Binomial: A polynomial consisting of **two** terms
Trinomial: A polynomial consisting of **three** terms

Multiplication of Polynomials

Example 1: $(3x^5)(-2x^9)$

Example 2: $5x^3(2x^5 - 4x^3 - x + 8)$

Example 3: Multiply and simplify.

a. $(x + 3)(x + 4)$

b. $(m - 5)(m - 6)$

c. $(2d - 4)(3d + 5)$

d. $(x - 2)(x^2 + 2x - 4)$

Example 4: Multiply and simplify

a. $(n + 5)^2$

b. $(3 - 2a)^2$

Example 5: Multiply and simplify.

a. $5(2x - 4)(3x + 2)$

b. $(x + 3)^2 - (x - 1)^2$

c. $(x - 2)^3$

Division of Polynomials

Variables represent nonzero quantities.

Example 6: $\dfrac{-6w^8}{30w^3}$

Example 7: $\dfrac{3x-6}{2}$

Example 8: $\dfrac{6x^3+2x^2-4}{4x}$

Example 9: $\dfrac{20a^2+35a-4}{-5a^2}$

Section 3.3: Evaluating Functions with Exponents

Example 1: Given the function, $f(x) = 2x^2$, evaluate each of the following.

a. $f(-5)$

b. $f\left(\frac{5}{6}\right)$

c. $f(10x + 1)$

d. $f(-2x^3)$

Example 2: Given the function $f(x) = -7x^2 + 4x - 8$, find a simplified expression for:

a. $f(x - 2)$

b. $f(x + h)$

Section 3.4: Operations on Functions

Basic Mathematical Operations
The basic mathematical operations are: addition, subtraction, multiplication, and division. When working with function notation, these operations will look like this:

Addition	Subtraction	Multiplication	Division
$f(x) + g(x)$	$f(x) - g(x)$	$f(x) \cdot g(x)$	$\frac{f(x)}{g(x)} = f(x) \div g(x)$ $g(x) \neq 0$

Example 1 (Addition and Subtraction): Given $f(x) = 3x - 6$ and $g(x) = 2x^2 + 7x + 5$, determine each of the following.

$f(7) + g(5)$

$f(3) - g(2)$

$f(x) + g(x)$

$f(x) - g(x)$

$g(x) - f(x)$

Example 2 (Multiplication): Given $f(x) = 3x + 2$ and $g(x) = 2x^2 + 3x - 1$, determine $f(x) \cdot g(x)$.

Example 3 (Division): For each of the following, determine $\frac{f(x)}{g(x)}$. Use only positive exponents in your final answer.

a. $f(x) = 10x^4 + 3x^2$ and $g(x) = 2x^2$

b. $f(x) = -12x^5 + 8x^2 + 5$ and $g(x) = -4x^2$

Example 4: The functions $f(x)$ and $g(x)$ are defined by the tables below.

x	–3	–2	0	1	4	5	8	10	12
$f(x)$	8	6	3	2	5	8	11	15	20

x	0	2	3	4	5	8	9	11	15
$g(x)$	1	3	5	10	4	2	0	–2	–5

a. $f(1)$

b. $g(9)$

c. $f(0) + g(0)$

d. $g(5) - f(8)$

e. $f(0) \cdot g(3)$

Section 3.5: Composition of Functions

Function Composition is the process by which the OUTPUT of one function is used as the INPUT for another function. Two functions *f*(x) and *g*(x) can be composed as follows:

f(*g*(x)), where the function *g*(x) is used as the INPUT for the function *f*(x).
NOTE: This can also be written as "$f \circ g(x)$"

OR

g(*f*(x)), where the function *f*(x) is used as the INPUT for the function *g*(x).
NOTE: This can also be written as "$g \circ f(x)$"

Example 1: The functions *f* (*x*) and *g*(*x*) are defined by the tables below.

x	1	2	3	4	5	6	7	8	9	10
f (*x*)	4	11	10	8	6	5	8	2	6	9

x	1	2	3	4	5	6	7	8	9	10
g(*x*)	3	8	4	10	2	5	11	0	4	1

f(*g*(5)) =

f(*g*(1)) =

g(*f*(4)) =

f(*f*(1)) =

Example 2: Let A(*x*) = 2*x* + 1 and B(*x*) = 3*x* – 5. Evaluate each of the following.

A(B(*x*)) =

B(A(*x*)) =

A(B(4)) =

B(A(4)) =

Example 3: Find $f \circ g$ and $g \circ f$, given $f(x) = x^2 - 2x + 3$ and $g(x) = 2x + 1$.

$f \circ g$ $g \circ f$

Function Decomposition

In some cases, it is desirable to **decompose** a function – to write it as a composition of two simpler functions.

In previous examples, you were given 2 functions and asked to **compose** them to produce a 3rd function. **Function Decomposition** is the reverse: you are given the 3rd function and are asked to write possible equations for the original 2 functions.

Example 4: Given h(x) =$\sqrt{5x-1}$, write h(x) as a composite function in the form $(f \circ g)(x)$.

Section 3.6: One-to-One Functions

A **function** is a relation in which every input has exactly one output.
Every x has exactly one corresponding y

A **one-to-one function** is a function in which every input has exactly one *unique* output.
Every y has exactly one corresponding x

Example 1: Determine if the following tables are one-to-one functions.

x	y
-4	3
-2	5
1	9
3	3

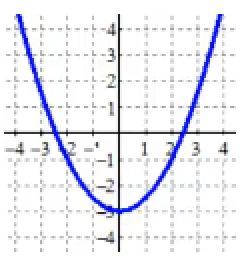
x	y
-4	-5
-2	2
1	0
3	2

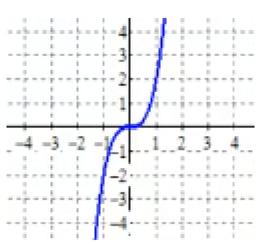
x	y
-4	3
-2	4
1	5
3	8

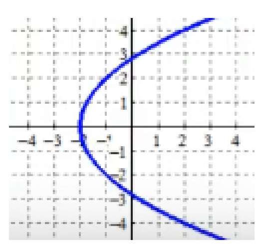
Graphically, the **Vertical Line Test** is a way to determine if a relation is a function.

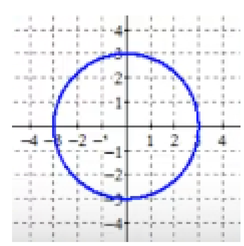
Graphically, the **Horizontal Line Test** is a way to determine if a function is a one-to-one function..

Example 2: Which graphs represent one-to-one functions?









Section 3.7: Inverse Functions

In general, a function and its inverse "undo" each other.

Notation: The inverse of f is f^{-1}

Properties of Inverse Functions:

If the function f is a one-to-one function, then the inverse of f is f^{-1} and the following is true:

- The input of f is the output of f^{-1} and vice versa.
- The domain of f is the range of f^{-1} and vice versa
- The graphs of f and f^{-1} are symmetric about the line $y = x$
- Composition: $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$
- NOTE: An inverse function is not a "reciprocal" function. $f^{-1}(x) \neq \frac{1}{f(x)}$

Example 1: Assume f is a one-to-one function. Determine the function values.

If $f(2) = 5$, then $f^{-1}(5) =$

If $f^{-1}(-3) = -2$, then $f(-2) =$

Example 2: Given H = f(t) where H is the height in meters of an object and t is the time in seconds since it was launched. Interpret the notation:

$f(10) = 16$

$f^{-1}(10) = 16$

Example 3: Determine each value using the table provided.

x	$f(x)$
-4	2
-3	-4
-2	0
-1	-2
0	1
1	3
2	-1
3	4
4	-3

x	$f^{-1}(x)$

$f(2)$ = ________

If $f(x) = -3$, then x = ________

$f^{-1}(-2)$ = ________

If $f^{-1}(x) = 1$, then x = ________

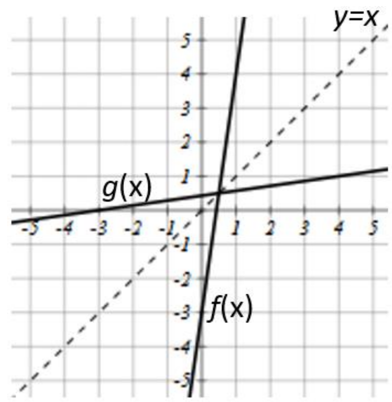
Section 3.8: Composing a Function with its Inverse

If f and g are inverses, then $f \circ g = f(g(x)) = x$ and $g \circ f = g(f(x)) = x$

Example 1: Determine if $f(x) = 7x - 3$ and $g(x) = \frac{x+3}{7}$ are inverses.

$f \circ g$

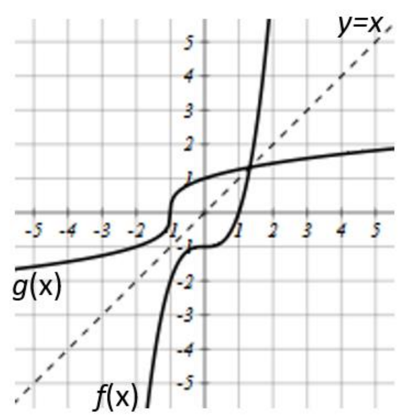
$g \circ f$



Example 2: Determine if $f(x) = x^3 - 1$ and $g(x) = \sqrt[3]{x+1}$ are inverses.

$f \circ g$

$g \circ f$



Section 3.9: Finding Inverses Algebraically

A function and its inverse "undo" each other.

$f(x) = y \leftrightarrow f^{-1}(y) = x$

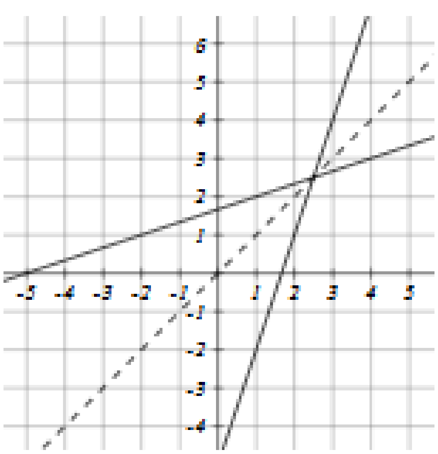
How to determine an inverse function

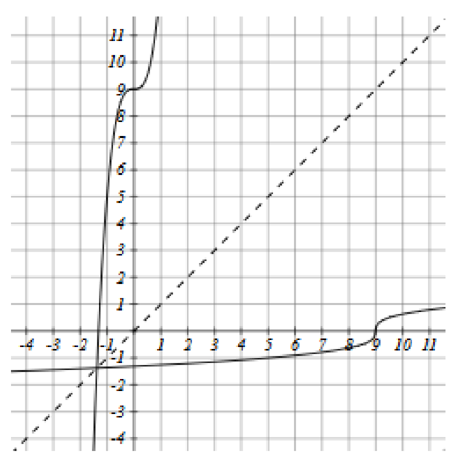
1. Determine if the function is one-to-one
2. Replace "*f*(x)" with "y"
3. Interchange the x and y variables. This new function is the inverse function.
4. If the result is an equation, solve the equation for y. Replace y with f^{-1}, symbolizing the inverse function or the inverse of *f*.
5. CHECK! Graph both functions with the line y=x to make sure they are symmetric about the y=x line.

Example 1: Find the inverse function for each one-to-one function.

$f(x) = 3x - 5$

$f(x) = 4x^3 + 9$



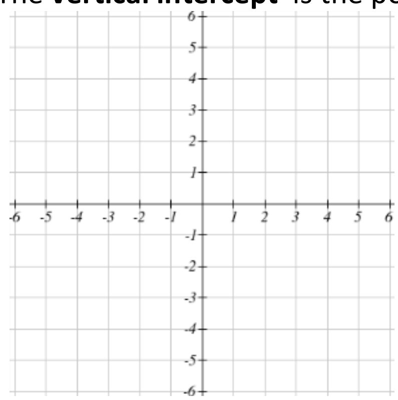


Unit 4: Rates of Change and Behavior of Graphs

Section 4.1: Characteristics of Graphs

Vertical and Horizontal Intercepts

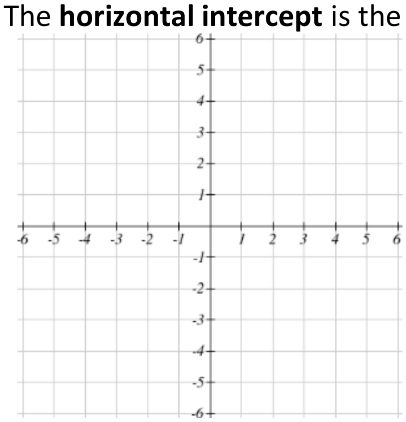
The **vertical intercept** is the point at which the graph crosses the vertical axis.



The input value of the vertical intercept is always___________

The coordinates of the vertical intercept will be ___________

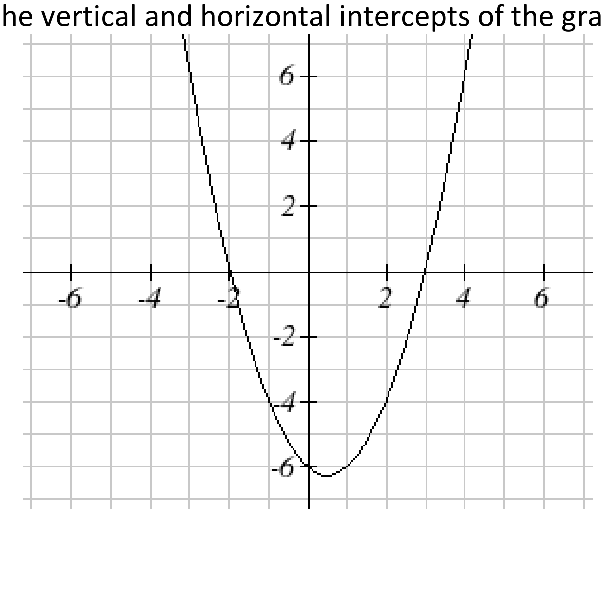
The **horizontal intercept** is the point at which the graph crosses the horizontal axis.



The output value of the horizontal intercept is always______

The coordinates of the horizontal intercept will be _________

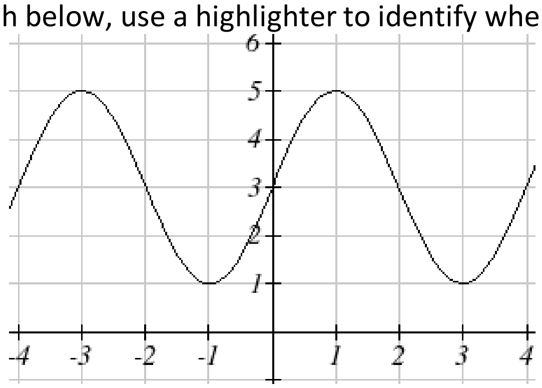
Example 1: Identify the vertical and horizontal intercepts of the graph below.



Behavior of Graphs
A graph is **increasing** if as the inputs increase, the outputs increase. A graph is **decreasing** if as the inputs increase, the outputs decrease. A graph is **constant** if as the inputs increase, the outputs do not change.

Increasing	Decreasing	Constant

Example 2: On the graph below, use a highlighter to identify where the graph is **increasing.**



Section 4.2: Behavior and Extrema

A function is **increasing** on an interval if the function values increase as the inputs increase. More formally, a function is increasing if f(b) > f(a) for any two input values a and b in the interval with b>a.

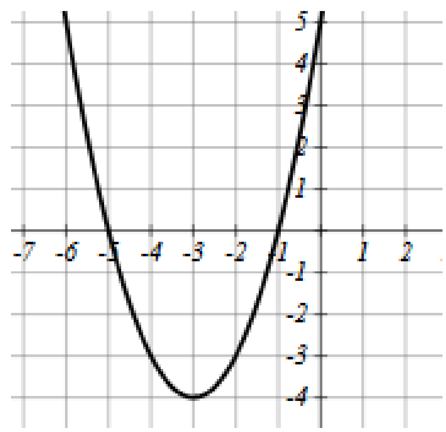
A function is **decreasing** on an interval if the function values decrease as the inputs increase. More formally, a function is decreasing if f(b) < f(a) for any two input values a and b in the interval with b>a.

A point where a function changes from increasing to decreasing is called a **local maximum.**

A point where a function changes from decreasing to increasing is called a **local minimum.**

Together, local maxima and minima are called the local extrema, or local extreme values, of the function.

Example 1: Consider the function graphed at right.



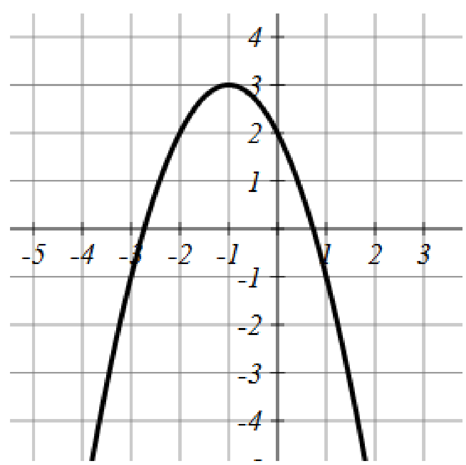
The function is increasing on the open interval:

The function is decreasing on the open interval:

The function has a maximum or minimum

of __________ at x = ___________

Example 2: Consider the function graphed at right.



The function is increasing on the open interval:

The function is decreasing on the open interval:

The function has a maximum or minimum

of __________ at x = ___________

Section 4.3: Average Rates of Change

Given any two points (x_1, $f(x_1)$) and (x_2, $f(x_2)$), the **<u>average rate of change</u>** between the points on the interval x_1 to x_2 is determined by computing the following ratio:

$$\text{Average rate of change} = \frac{\text{Change in Output}}{\text{Change in Input}} = \frac{\Delta\text{output}}{\Delta\text{input}} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Units for the Average Rate of Change are always: $\dfrac{\text{output units}}{\text{input unit}}$,

Units are read as “output units *per* input unit”

“ D ” notation denotes change so average rate of change can be thought of as “change in output divided by change in input”

The average rate of change of an **increasing function** is positive.

The average rate of change of a **decreasing function** is negative.

A function is ***linear*** if the average rate of change is constant

Example 1: Over a 7-week period, the price of an MP3 player dropped steadily, from $184 to $100. What was the average weekly change in the price of the MP3 player over this period?

Example 2: The function C(t) below gives the average cost, in dollars, of a gallon of gasoline t years after 2000. Find the average rate of change for each of the following time intervals. Interpret the meaning of your answers.

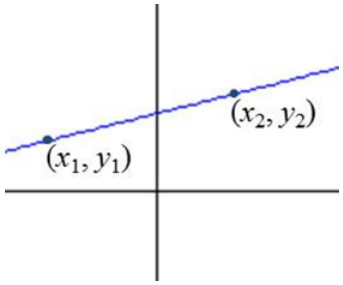
t	2	3	4	5	6	7	8	9
C(t)	1.47	1.69	1.94	2.30	2.51	2.30	3.01	2.14

a) 2003 to 2008

b) 2006 to 2009

c) 2005 to 2007

Section 4.4: Slope and Behavior of a Line

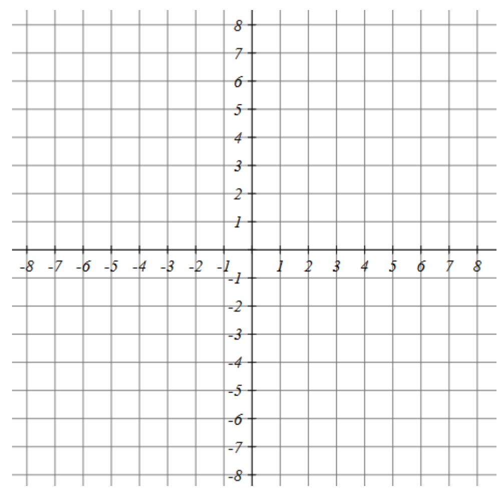


$$m = \text{Slope} = \frac{\text{Change in OUTPUT}}{\text{Change in INPUT}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

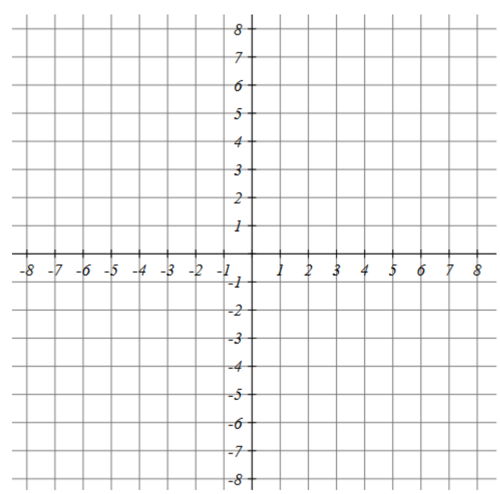
Slope	Behavior
$m > 0$	Increasing
$m < 0$	Decreasing
$m = 0$	Horizontal
m is undefined	Vertical

Example 1: Find the slope of the line passing through the given points. Then determine if the line would be increasing, decreasing, horizontal, or vertical.

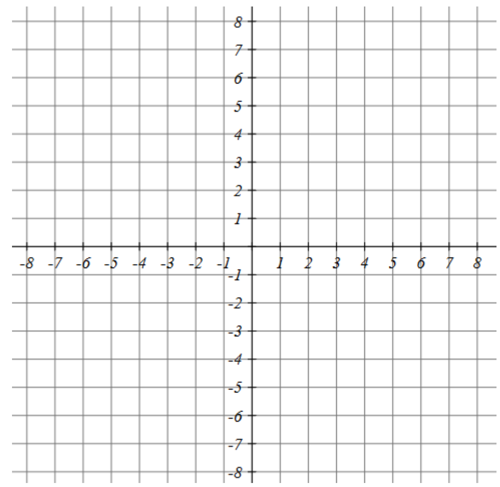
(-2, 3) and (4, -7)



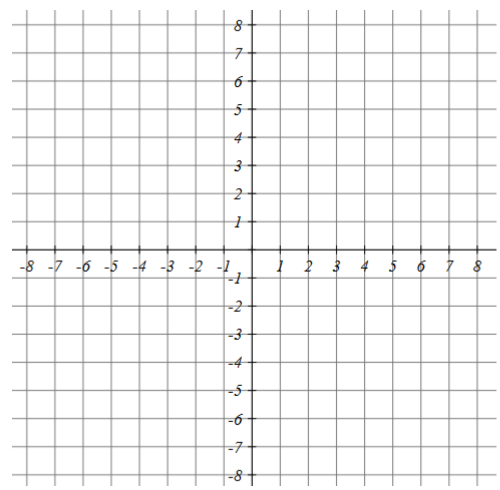
(2, 6) and (2, -3)



(-4, -2) and (6, 3)

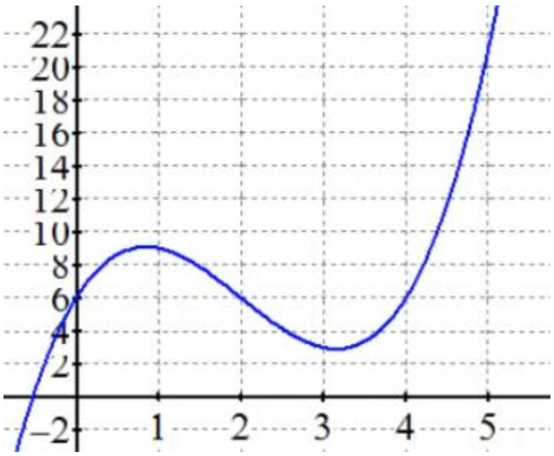


(-3, -3) and (1, -3)



Section 4.5: Average Rates of Change of Functions and Graphs

Example 1: Use the graph to determine the Average Rate of Change over the given intervals.



Average Rate of Change from x = 0 to x = 1

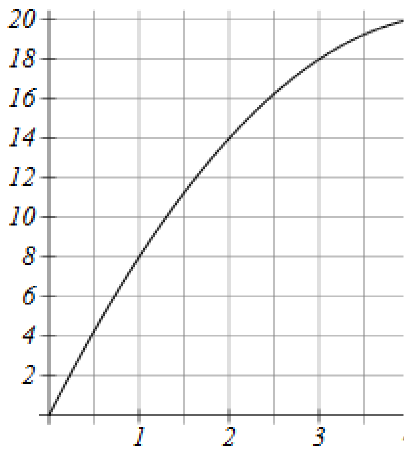
Average Rate of Change from x = 2 to x = 5

Example 2: Find the average rate of change of $f(x) = -x^2 + \frac{9}{x}$ on the interval [1, 6].

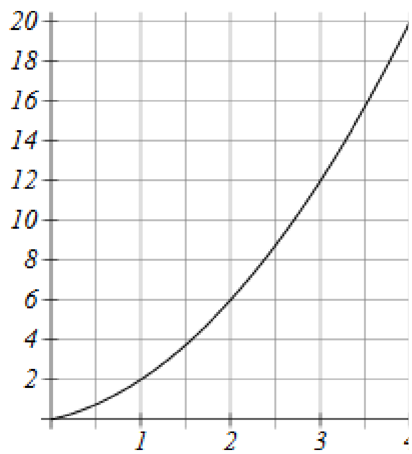
Section 4.6: Comparing Rates of Change

Example 1: The total sales, in thousands of dollars, for three companies over 4 weeks are shown below. Total sales for each company are clearly increasing, but in very different ways. To describe the behavior of each function, we can compare its average rate of change on different intervals.

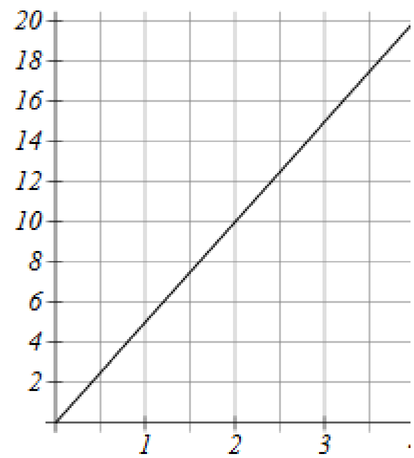
Company A: A(t)



Company B: B(t)



Company C: C(t)



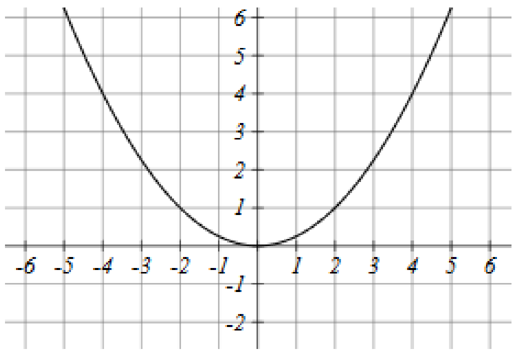
AROC	A(t)	B(t)	C(t)
[0,1]			
[1,2]			
[2,3]			
[3,4]			

What do you notice about the values above for each function? What do these values tell you about each function?

Section 4.7: Concavity

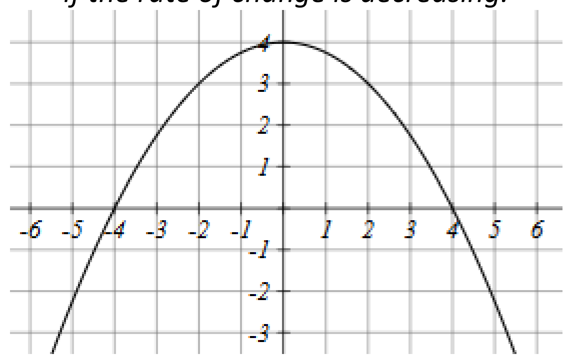
Example 1:

f(x) is **concave up**
if the rate of change is increasing.



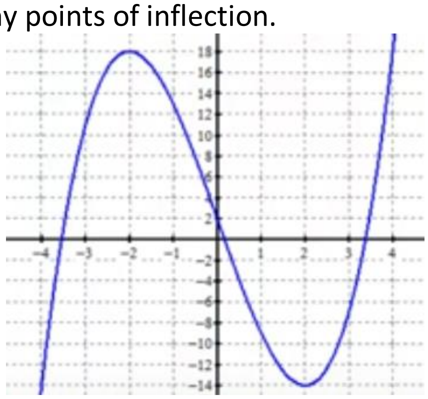
m = ____________________

f(x) is **concave down**
if the rate of change is decreasing.



m = ____________________

Example 2: Determine the intervals for which the function is concave up and concave down. Determine any points of inflection.



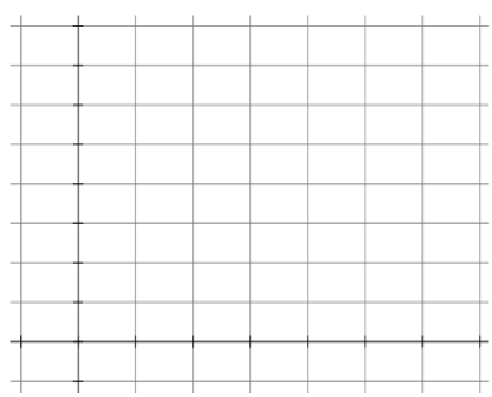
Concave Up:

Concave Down:

Points of Inflecton:

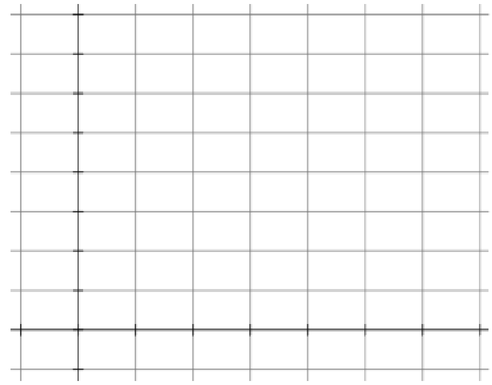
Example 3: Use the table to determine if the function is increasing or decreasing and whether the function is concave up or concave down.

x	f(x)
0	120
1	103
2	91
3	84
4	78
5	74



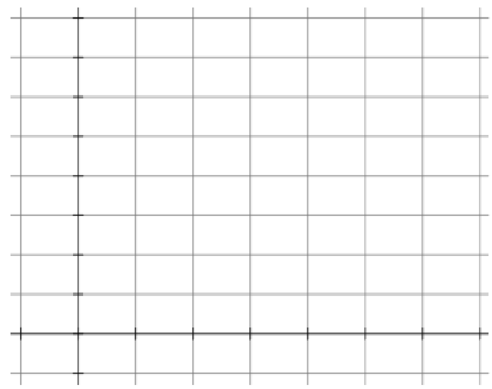
Example 4: Use the table to determine if the function is increasing or decreasing and whether the function is concave up or concave down.

x	f(x)
0	115
1	109
2	98
3	80
4	57
5	32



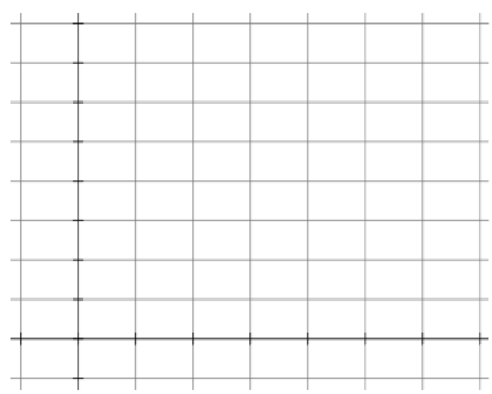
Example 5: Use the table to determine if the function is increasing or decreasing and whether the function is concave up or concave down.

x	f(x)
0	4
1	11
2	23
3	41
4	66
5	98



Example 6: Use the table to determine if the function is increasing or decreasing and whether the function is concave up or concave down.

x	f(x)
0	16
1	59
2	94
3	121
4	136
5	142



Section 4.8: Secant Lines

A **secant line** is a straight line connecting any two points on a function.

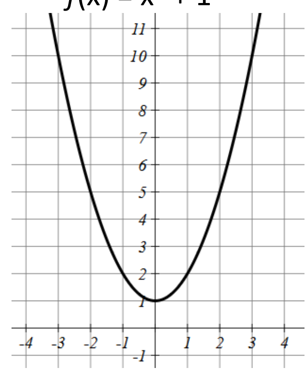
The **slope of the secant line** is equivalent to the **average rate of change** of the function between those two points.

Example 1: Find the average rate of change of $f(x) = x^2 + 1$ on the following intervals, and draw the corresponding secant lines on the graphs provided.

Average rate of change on the interval $[x_1, x_2]$: $\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{\text{"rise"}}{\text{"run"}}$

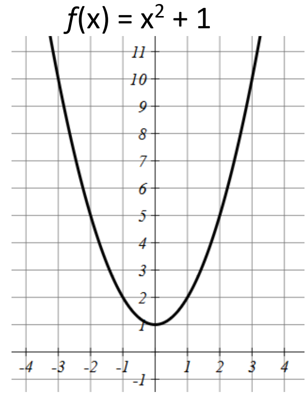
a. [-2, 3]

$f(x) = x^2 + 1$



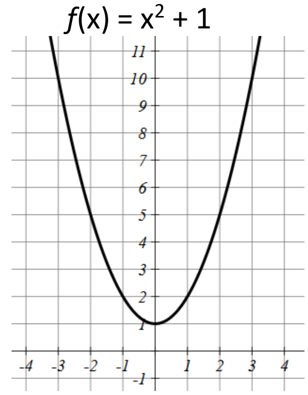
b. [-2, 2]

$f(x) = x^2 + 1$



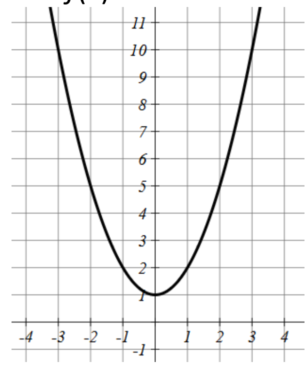
c. [-2, 1]

$f(x) = x^2 + 1$



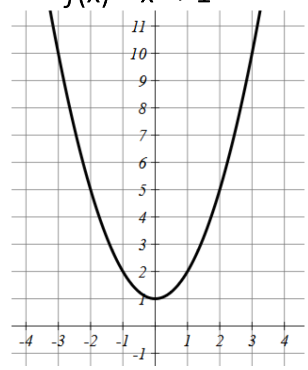
d. [-2, 0]

$f(x) = x^2 + 1$



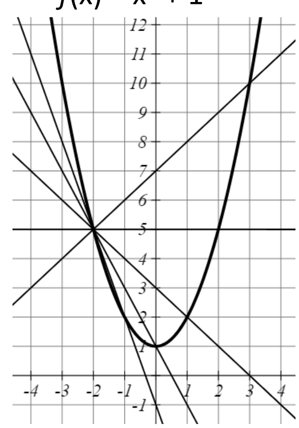
e. [-2, -1]

$f(x) = x^2 + 1$



f. [-2, -2 + h]

$f(x) = x^2 + 1$



Section 4.9: The Difference Quotient

The Difference Quotient gives the average rate of change of a function on the interval [x, x+h].

$$\text{Difference Quotient: } = \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(x+h)-f(x)}{h}, \qquad h \neq 0$$

Example 1: Find $\frac{f(x+h)-f(x)}{h}$, given $f(x) = 2x - 5$.

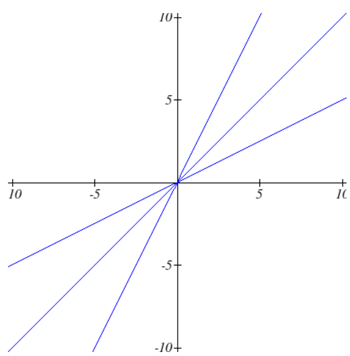
Example 2: Find $\frac{f(x+h)-f(x)}{h}$, given $f(x) = x^2 - 3x + 4$.

Unit 5: Linear Functions

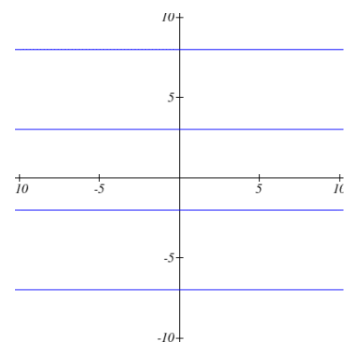
Section 5.1: Linear Functions

A linear function is a function that fits the form:

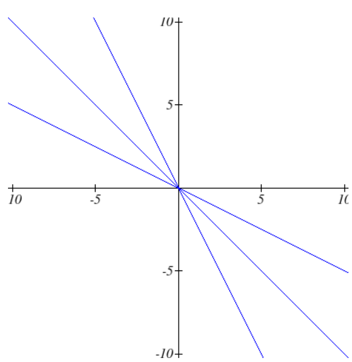
A linear function can be graphically represented by a ________________________________



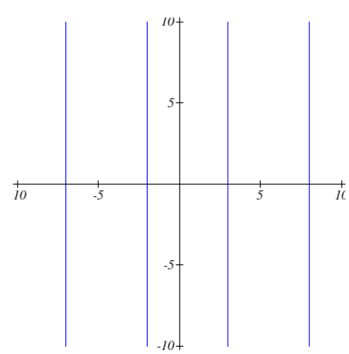
Increasing Linear Function
Slope > 0



Constant Function
Slope = 0

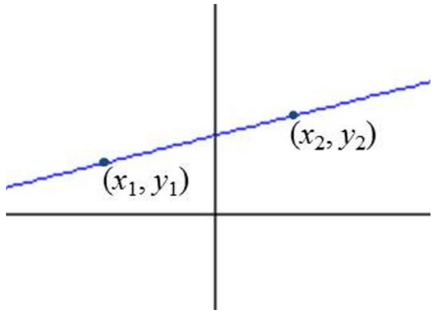


Decreasing Linear Function
Slope < 0



Not a Function
Slope is Undefined (No Slope)

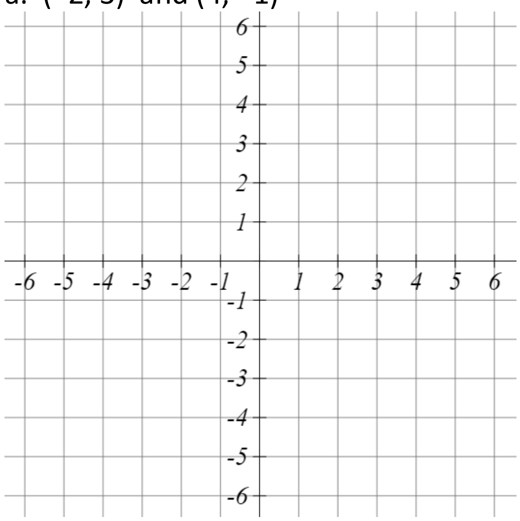
$$m = \text{Slope} = \frac{\text{Change in OUTPUT}}{\text{Change in INPUT}} = \frac{\Delta \text{ OUTPUT}}{\Delta \text{ INPUT}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$



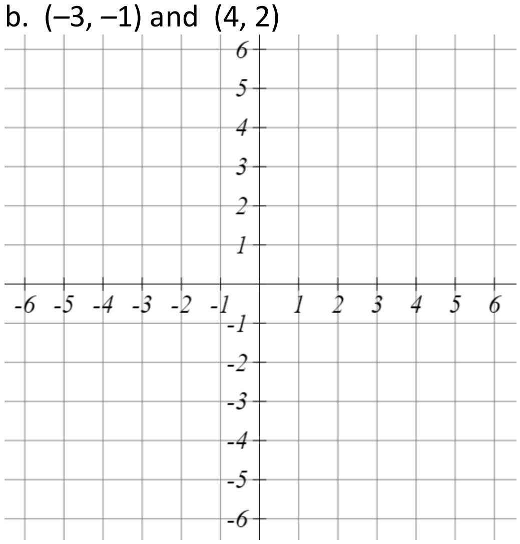
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1: Determine the slope for each of the following:

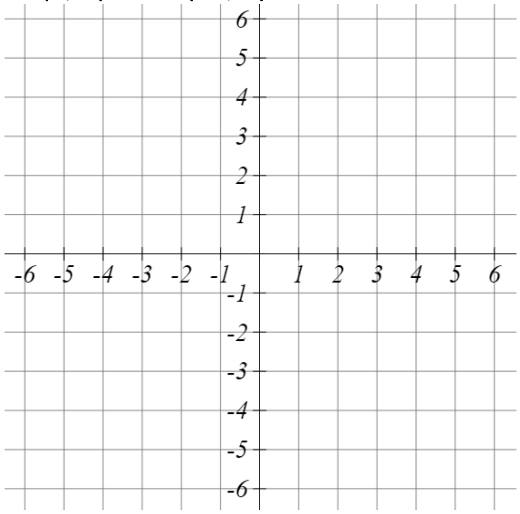
a. (–2, 3) and (4, –1)



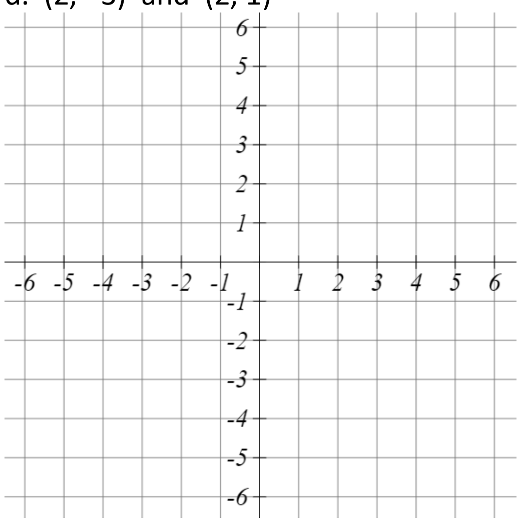
b. (–3, –1) and (4, 2)



c. (3, 2) and (–1, 2)



d. (2, –3) and (2, 1)



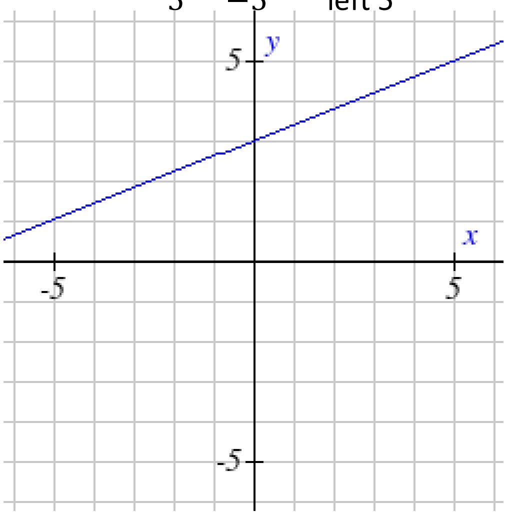
Section 5.2: Graphing Linear Functions

USING THE SLOPE TO GRAPH A LINEAR FUNCTION

$$m = \text{Slope} = \frac{\text{Change in OUTPUT}}{\text{Change in INPUT}} = \frac{\text{Vertical Change}}{\text{Horizontal Change}} \rightarrow \frac{\updownarrow}{\leftrightarrow}$$

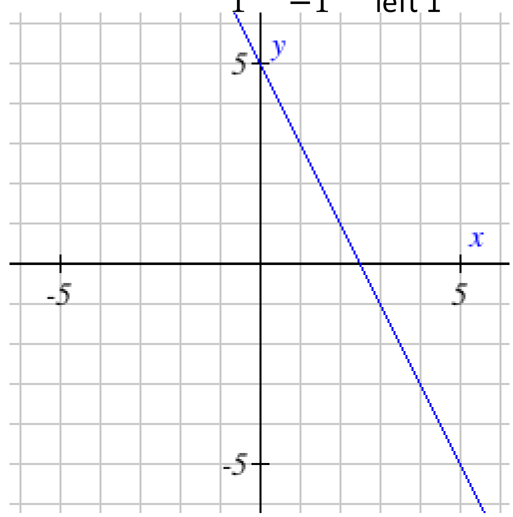
$$m = \frac{2}{5} \rightarrow \frac{\text{up } 2}{\text{right } 5}$$

$$m = \frac{2}{5} = \frac{-2}{-5} \rightarrow \frac{\text{down } 2}{\text{left } 5}$$



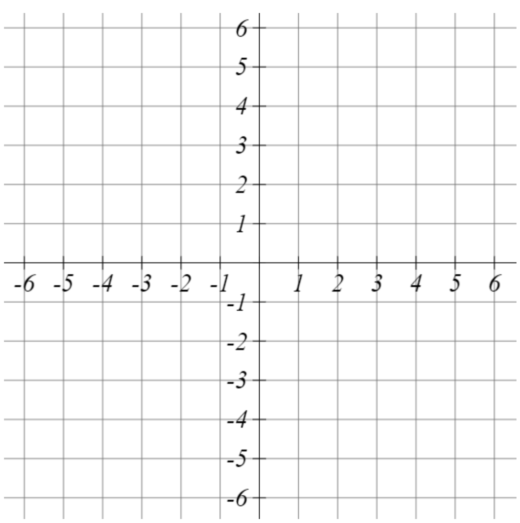
$$m = -2 = -\frac{2}{1} = \frac{-2}{1} \rightarrow \frac{\text{down } 2}{\text{right } 1}$$

$$m = -2 = -\frac{2}{1} = \frac{2}{-1} \rightarrow \frac{\text{up } 2}{\text{left } 1}$$

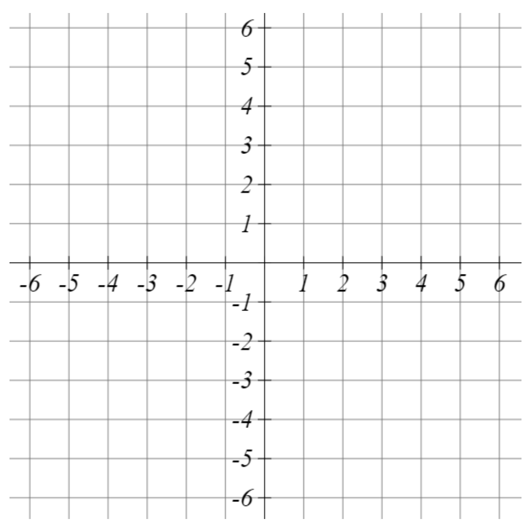


Example 1: Draw an accurate graph for each of the following

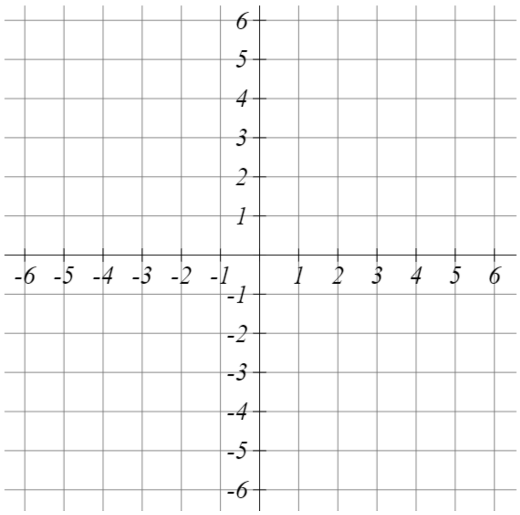
a. (–2, –3) slope $\frac{1}{2}$



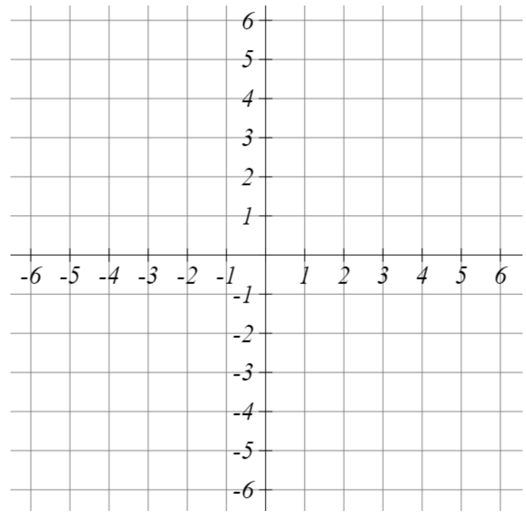
b. (0, –1) slope $-\frac{2}{3}$



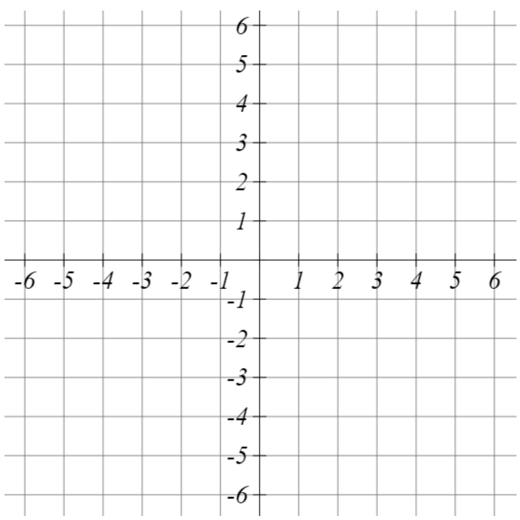
c. (2, 1) slope 3



d. (1, –4) slope 0



e. (5, 2) undefined slope



Section 5.3: The Equation of a Linear Function

A linear function is a function that fits the form: $f(x) = mx + b$ or $f(x) = b + mx$

b is the initial value; vertical intercept (0,b)

$$m = \text{Slope} = \frac{\text{Change in OUTPUT}}{\text{Change in INPUT}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Slope	Behavior
$m > 0$	Increasing
$m < 0$	Decreasing
$m = 0$	Horizontal
m is undefined	Vertical

Example 1: Fill in the table below.

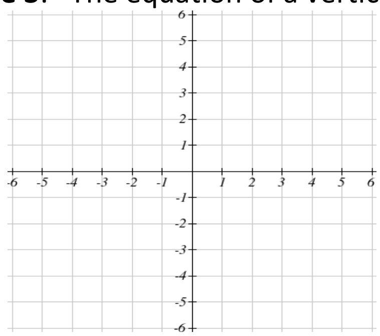
Equation	Slope	I, D, H, V	Vertical Intercept
$y = 3x + 5$			
$y = 8 - x$			
$y = 2x$			
$y = -8$			

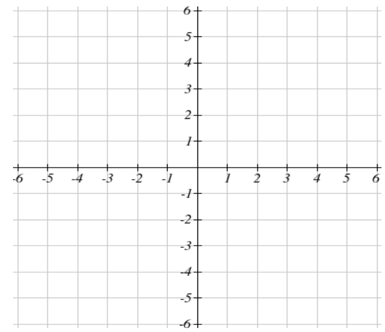
Example 2: Determine the *horizontal* intercepts of each of the following.

$y = 3x + 5$ $y = 8 - x$ $y = 2x$ $y = -8$

To find a horizontal intercept: ____

Example 3: The equation of a vertical line



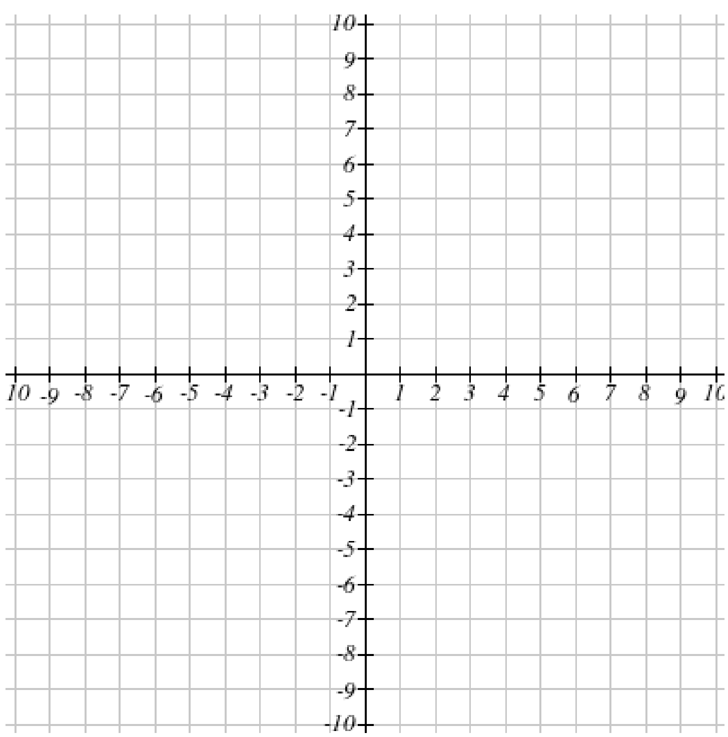


Example 4: Draw an **accurate** graph of the function $f(x) = 4 - 3x$.

Slope: ____________

Vertical Intercept: ____________

Horizontal Intercept: __________



To find the Horizontal Intercept:

____

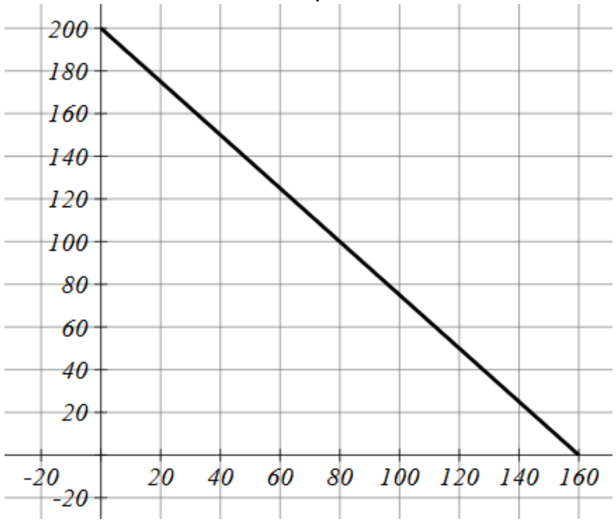
____

Two additional points on the line:

____________ ______________

Slope-Intercept Form
$f(x) = mx + b$
$f(x) = b + mx$

Example 5: The function $A(m) = 200 - 1.25m$ represents the balance in a bank account (in thousands of dollars) after m months.



a. Identify the slope of this linear function and interpret its meaning in a complete sentence.

b. Identify the vertical intercept. Write it as an ordered pair and interpret its practical meaning in a complete sentence.

Ordered Pair: _______________

c. Determine the horizontal intercept of this linear function. Write it as an ordered pair and interpret its practical meaning in a complete sentence.

Ordered Pair:_________________

d. Determine $A(12)$. Write your answer as an ordered pair and interpret its practical meaning in a complete sentence.

Ordered Pair:_________________

e. How long will it take for the balance in this account to reach $80,000? Write the corresponding ordered pair.

Ordered Pair:_________________

Section 5.4: Writing the Equation of a Line in Slope-Intercept Form

Slope-Intercept Form $y = mx + b$

Example 1: Give the equation of the line in slope-intercept form

a. With vertical intercept (0, 2) and slope –9

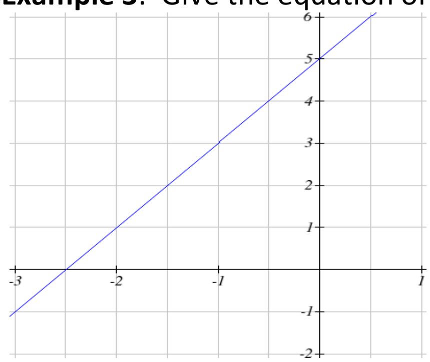
b. Passing through (2, 3) with slope –5

c. Passing through (2, 6) and (4, 16)

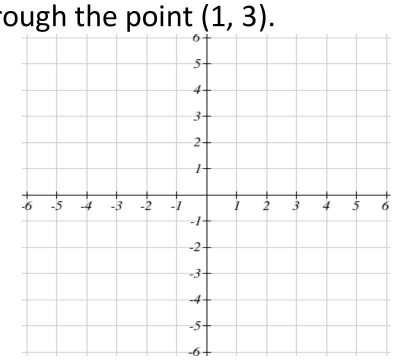
Example 2: Give the equation of the linear function that would generate the following table of values. Use your calculator to check.

x	$f(x)$
–5	238
–3	174
–1	110
1	46
7	–146
12	–306

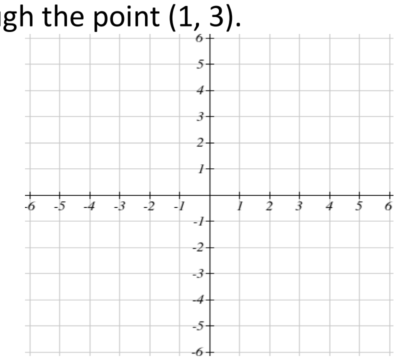
Example 3: Give the equation of the linear function shown below.



Example 4: Give the equation of the horizontal line passing through the point (1, 3).



Example 5: Give the equation of the vertical line passing through the point (1, 3).



Example 6: When a new charter school opened in 2005, there were 520 students enrolled. Write an equation for N representing the number of students attending the school t years after 2005, assuming that the student population:

Increased by 32 students per year: ____________________________________

Decreased by 48 students per year: ____________________________________

Increased by 20 students every 2 years: ____________________________________

Decreased by 28 students every 4 years: ____________________________________

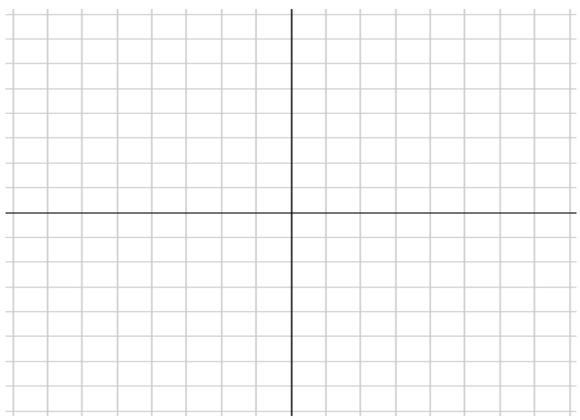
Remained constant (did not change): ____________________________________

Increased by 10 students every semester: ____________________________________

Section 5.5: Parallel and Perpendicular Lines

Parallel Lines

The slopes of Parallel Lines are ____



Slope-Intercept Form

$y = mx + b$

$f(x) = mx + b$

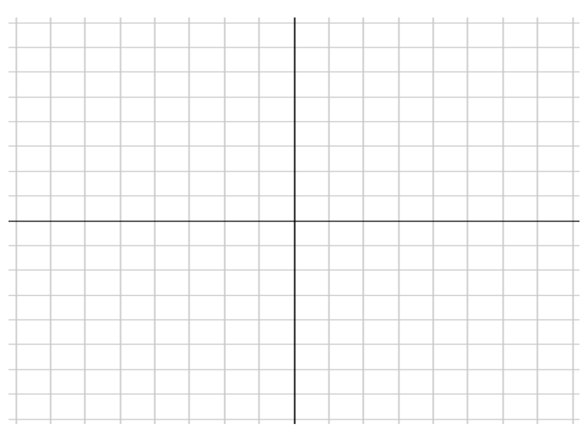
m = slope

b = vertical intercept $(0,b)$

Example 1: Give the equation of the line passing through the point (8, 3) that is **parallel** to the line $y = -2x + 3$.

Perpendicular Lines

The slopes of perpendicular lines are ____



If Line 1 and Line 2 are perpendicular to each other, then

Slope of Line 1	Slope of Line 2
$\frac{2}{3}$	
5	
–8	
$-\frac{4}{5}$	

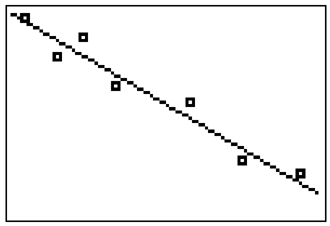
Negative (Opposite) Reciprocals

$\frac{a}{b}$ and $-\frac{b}{a}$

Example 2: Give the equation of the line passing through the point (8, 3) that is **perpendicular** to the line $y = -2x + 3$.

Section 5.6: Linear Regression

Any data set can be modeled by a linear function even those data sets that are not exactly linear. In fact, most real world data sets are not exactly linear and our models can only *approximate* the given values. The process for writing linear models for data that are not perfectly linear is called *linear regression*. Statistics classes teach a lot more about this process. In this class, you will be introduced to the basics. This process is also called *finding line of the best fit*.

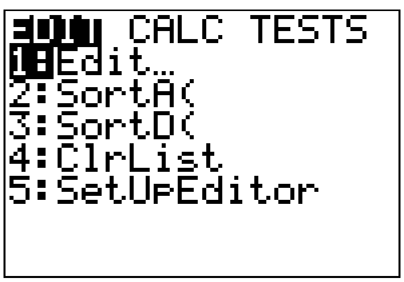


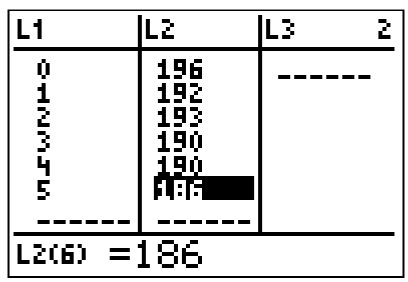
Consider the data set from the previous weight-loss problem:

Time, t, in weeks	0	1	2	3	4	5
Weight, W(t) in pounds	196	192	193	190	190	186

Step 1: Enter the Data into your Graphing Calculator

Press STAT then select option 1:Edit under EDIT menu. Clear lists, then enter the values.

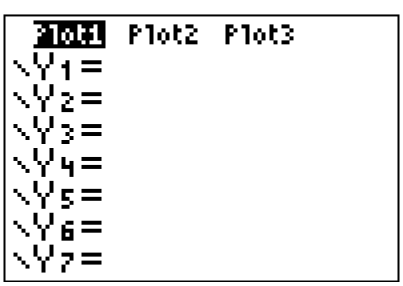


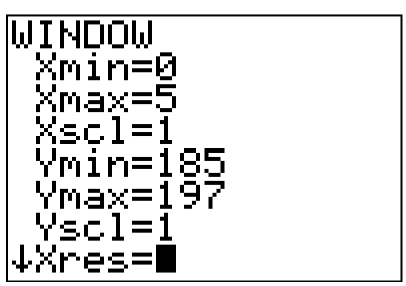


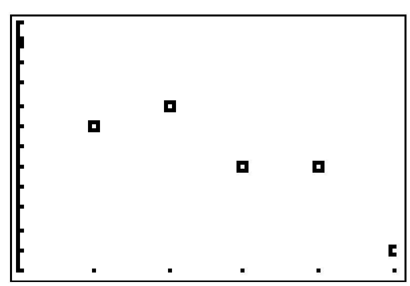
**NOTE *If you ever accidentally DELETE a column, then go to STAT>5: SetUpEditor>ENTER. When you go back to STAT, your column should be restored.*

Step 2: Turn on your Stat Plot and Graph the Data in an Appropriate Viewing Window

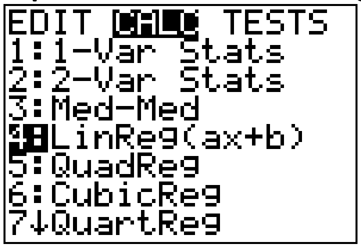
(Refer to previous example for help)





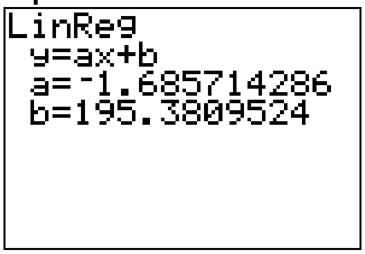


Step 3: Access the Linear Regression section of your calculator



- Press STAT
- Scroll to the right one place to CALC
- Scroll down to 4:LinReg(ax+b)
- Your screen should look as the one at left

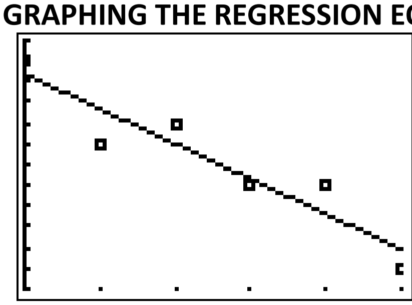
Step 4: Determine the linear regression equation



- Press ENTER twice in a row to view the screen at left
- The calculator computes values for slope (a) and y-intercept (b) in what is called the equation of best-fit for your data.
- Identify these values and round to the appropriate places. Let's say 2 decimals in this case.
  So, a = -1.69 and b = 195.38
- Now, replace the a and b in y = ax + b with the rounded values to write the actual equation:
  $y = -1.69x + 195.38$
- To write the equation in terms of initial variables, we would write $W = -1.69t + 195.38$
- In function notation, $W(t) = -1.69t + 195.38$

Once we have the equation figured out, it's nice to graph it on top of the scatterplot to see how things match up.

GRAPHING THE REGRESSION EQUATION ON TOP OF THE STAT PLOT



- Enter the Regression Equation with rounded values into Y=
- Press GRAPH
- You can see from the graph that the "best fit" line does not hit very many of the given data points. But, it will be the most accurate linear model for the overall data set.

IMPORTANT NOTE: When you are finished graphing your data, TURN OFF YOUR PLOT1. Otherwise, you will encounter an INVALID DIMENSION error when trying to graph other functions. To do this:

- Press Y=
- Use your arrow keys to scroll up to Plot1
- Press ENTER
- Scroll down and Plot1 should not be highlighted

Example 1: The function $f(n)$ is defined in the following table.

n	0	2	4	6	8	10	12
$f(n)$	23.76	24.78	25.93	26.24	26.93	27.04	27.93

a) Based on **this table**, determine $f(6)$. Write the specific ordered pair associated with this result.

b) Use your graphing calculator to determine the equation of the regression line for the given data. Round to three decimals as needed.

The regression equation in $f(n) = an + b$ form is: ___________________________

c) Use your graphing calculator to generate a scatterplot of the data *and* regression line on the same screen. You must use an appropriate viewing window. In the space below, draw what you see on your calculator screen, and write down the viewing window you used.

Xmin=____________

Xmax=____________

Ymin=____________

Ymax=___________

d) Using your **regression equation**, determine $f(6)$. Write the specific ordered pair associated with this result.

e) Your answers for a) and d) should be different. Explain why this is the case.

Common Calculator Errors

<u>SYNTAX</u> – Check your previous entry for syntax errors. Look for extra (or missing) parentheses. Look to see if you may have typed in a “-“ instead of a “–“ or vice versa.

<u>WINDOW RANGE</u> – Check your window settings. You may have an xmin that is larger than the xmax or a ymin that is larger than the ymax.

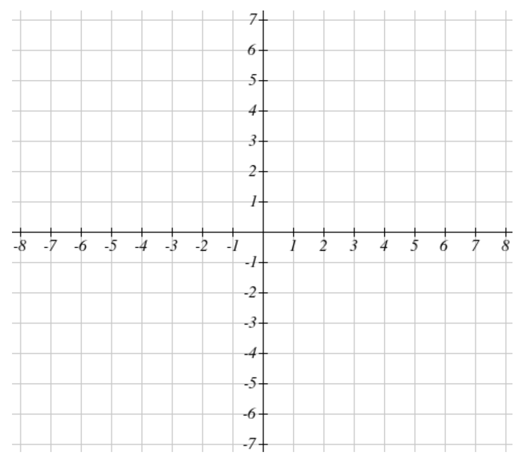
<u>DIM MISMATCH</u> – Check your lists (in STAT). They may be blank, or one list is longer than the other.

<u>NO SIGN CHNG</u> – In order to use the intersection method for solving equations, the intersection point must appear on your calculator screen. Adjust your window settings so that the intersection point appears on the screen.

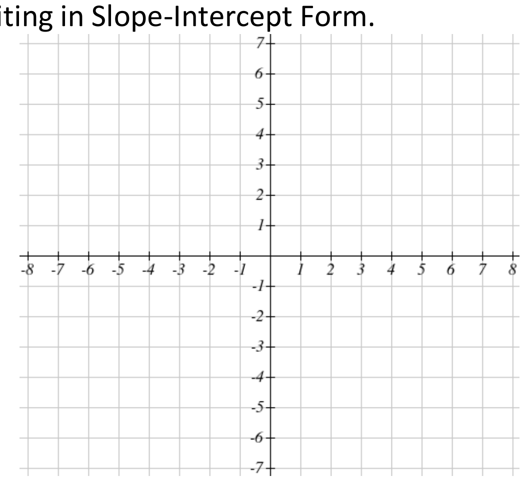
Section 5.7: Point-Slope Form of a Line

Slope-Intercept Form of a Linear Equation	**Point-Slope Form of a Linear Equation**
$y = mx + b$	$y - y_1 = m(x - x_1)$
x = input, y = output m = slope b = vertical intercept $(0, b)$	x = input, y = output m = slope (x_1, y_1) is a point on the line.

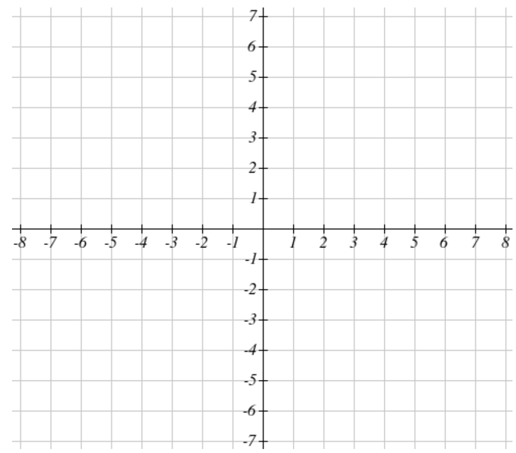
Example 1: Graph the line $y - 3 = -2(x + 4)$.



Example 2: Graph the line $y - 3 = -2(x + 4)$ by rewriting in Slope-Intercept Form.



Example 3: Graph the line $y + 3 = \frac{1}{2}(x - 1)$.



Example 4: Find the equation of the line with slope = ½ and passing through the point (6, -3).

Point-Slope Form
$y - y_1 = m(x - x_1)$

Slope-Intercept Form
$y = mx + b$

Example 5: Find the equation of the line containing the points (-1, 4) and (3, 5).

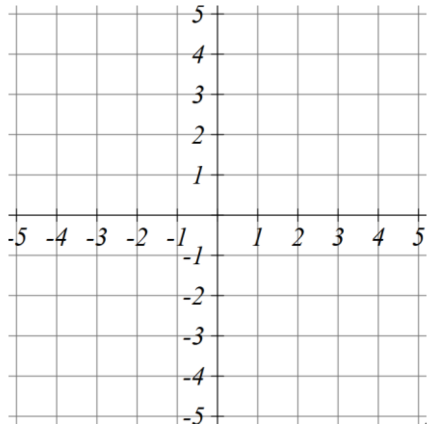
Point-Slope Form
$y - y_1 = m(x - x_1)$

Slope-Intercept Form
$y = mx + b$

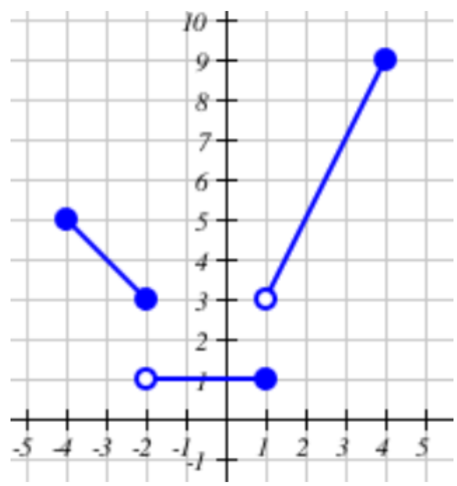
Section 5.8: Piecewise Functions

Example 1: Sketch the graph of the function

$$f(x) = \begin{cases} 3 & if \quad x \leq -2 \\ -x+1 & if \quad -2 < x \leq 1 \\ 1 & if \quad x > 1 \end{cases}$$



Example 2: Determine the formula for the graph shown below.

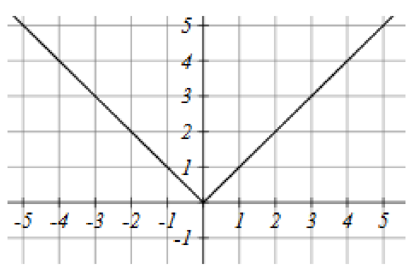


The Absolute Value Function

The Absolute Value Function can be defined as a piecewise function:

$$f(x) = |x| = \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0 \end{cases}$$

Domain: (-∞, ∞)
Range: [0, ∞)



Unit 6: Factoring and Solving Quadratic Equations

Section 6.1: Multiplication of Polynomials

Definitions
Polynomial: An algebraic expression composed of the sum of terms containing a single variable raised to a non-negative integer exponent.
Monomial: A polynomial consisting of **one** term
Binomial: A polynomial consisting of **two** terms
Trinomial: A polynomial consisting of **three** terms

Example 1: Multiply and simplify.

$(x + 7)(x + 5)$

$(2y – 3)(5y + 9)$

$(a – 2b)(7a – 4b)$

Example 2: Multiply and simplify.

$(x + 5)^2$

$(3x + 7)^2$

Example 3: Multiply and simplify.

$3x(x + 1)(x – 3)$

$(y – 2)(y^2 + 2y + 4)$

Section 6.2: The GCF Method

A **Quadratic Expression** is a second degree polynomial of the form $ax^2 + bx + c$, where a, b, and c are real number coefficients. Examples of quadratic expressions are:

- $3x^2 - x + 27$ Here, $a = 3$, $b = -1$, and $c = 27$
- $x^2 - 3$ Here, $a = 1$, $b = 0$, and $c = -3$
- $2x^2 + 5x$ Here, $a = 2$, $b = 5$, and $c = 0$
- $2.3x^2$ Here, $a = 2.3$, $b = 0$, and $c = 0$
- $3x + 8 - x^2$ Here, $a = -1$, $b = 3$, and $c = 8$

REVIEW: Factoring Whole Numbers
Worked Example: To factor 60, there are many possibilities, some of which are below: $60 = 1 \cdot 60$ (not very interesting, but true) $60 = 2 \cdot 30$ $60 = 3 \cdot 20$ $60 = 4 \cdot 3 \cdot 5$ $60 = 2 \cdot 2 \cdot 3 \cdot 5$ (This one is also called the *prime factorization* of 60 because it is made of only prime factors.

When we factor quadratic expressions, we use a similar process. This process involves factoring expressions such as $24x^2$. To factor $24x^2$ completely, we would first find the prime factorization of 24 and then factor x^2.

$$24 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \quad \text{and} \quad x^2 = x \cdot x$$

Putting these factorizations together, we obtain the following: $24x^2 = 2{\cdot}2{\cdot}2{\cdot}2{\cdot}3{\cdot}x{\cdot}x$

Example 1: Completely factor the quadratic expressions below. Check your final results.

a. $11a^2 - 4a$

b. $55w^2 + 5w$

Section 6.3: Factoring by Trial and Error ($a = 1$)

Factoring Quadratic Expressions of the form $x^2 + bx + c$ by Trial and Error
$x^2 + bx + c = (x + p)(x + q)$, where $b = p + q$ and $c = p \cdot q$

Worked Example: Completely factor the quadratic expression $x^2 + 5x - 6$.

Step 1: Look to see if there is a common factor in this expression. If there is, then use the GCF method to factor out the common factor.

The expression $x^2 + 5x - 6$ has no common factors.

Step 2: For this problem, $b = 5$ and $c = -6$. We need to identify p and q. These are two numbers whose product is –6 and sum is 5. A helpful method to identify p and q is to list different numbers whose product is –6 and then see (i.e. trial and error) which pair adds to 5.

Product = –6	Sum = 5
$-3 \cdot 2$	No
$3 \cdot -2$	No
$-1 \cdot 6$	YES
$1 \cdot -6$	No

Step 3: Write in factored form

$$x^2 + 5x - 6 = (x + (-1))(x + 6)$$

$$x^2 + 5x - 6 = \mathbf{(x - 1)(x + 6)}$$

Step 4: Always check your result!

$$(x - 1)(x + 6) = x^2 + 6x - x - 6$$

$$= x^2 + 5x - 6 \quad \text{CHECKS!}$$

Example 1: Completely factor the following quadratic expressions. Check your final results.

a. $a^2 + 7a + 12$ b. $w^2 + w - 20$ c. $x^2 - 36$

Example 2: Completely factor the following quadratic expressions.

a. $2x^2 + 10x - 12$ b. $6x^2 - 21x - 12$

Example 3: Completely factor the following expressions.

a. $2x^3 - 50x$ b. $3x^3 + 9x^2 - 84x$

Section 6.4: Factoring by Trial and Error when a ≠ 1

Trial and Error (a ≠ 1)

1. Place the factors of ax^2 in the 1^{st} positions of the two sets of parentheses that represent the factors
2. Place 2 possible factors of c into the 2^{nd} position of the parentheses
3. Find the inner and outer products of the 2 sets of parentheses
4. Keep trying different factors until the inner and outer products add to bx.

Example 1: Completely factor the following quadratic expressions.

a) $6x^2 – 5x – 4$

b) $15x^2 + 8x - 12$

Perfect Square Trinomials

To factor a perfect square trinomial we need to be able to recognize perfect square factors.

$$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$$

Example: $x^2 - 10x + 25 = (x - 5)(x - 5) = (x - 5)^2$

Example 2: Factor the following perfect square trinomials.

a) $4x^2 - 12x + 9$

b) $16x^2 + 56x + 49$

Section 6.5: Other Factoring Methods

Below are three different methods for factoring a quadratic expression: $ax^2 + bx + c$

Trial and Error (Reverse Foil)

1. Place the factors of ax^2 in the 1st positions of the two sets of parentheses that represent the factors
2. Place 2 possible factors of c into the 2nd position of the parentheses
3. Find the inner and outer products of the 2 sets of parentheses
4. Keep trying different factors until the inner and outer products add to bx.

Example 1: $3x^2 - 11x - 20$

Factoring by Grouping

1. Find the factors of ac that add to b
2. Write bx as a sum or difference using the factors from step 1
3. Divide or group the polynomial into halves
4. Factor out the GCF of the first half and second half
5. Factor out the common binomial factor

Example 2: $3x^2 - 11x - 20$

Bottoms Up

1. Multiply a and c. Then rewrite in the form: $x^2 + bx + ac$
2. Factor as you normally would by finding the factors of ac that add to b.
3. Divide the constants in each binomial factor by the original value of a.
4. Simplify the fractions formed.
5. If the simplified fraction does not have a denominator of 1, move the denominator to the coefficient of the variable.

Example 3: $3x^2 – 11x – 20$

Section 6.6: Solving Equations by Factoring

The Zero Product Property

If two numbers a and b are multiplied together and the resulting product is 0, then at least one of the numbers must be zero.

If $a \cdot b = 0$
then $a = 0$ or $b = 0$,
or both $a = 0$ and $b = 0$.

Example 1: Solve each equation.

a. $4x(x + 5)$

b. $(x - 2)(x + 7) = 0$

c. $x(x - 1)(6x + 11) = 0$

To solve a quadratic equation by factoring:

Step 1: Make sure the quadratic equation is in standard form: $ax^2 + bx + c = 0$.

Step 2: Write the left side in completely factored form.

Step 3: Apply the Zero Product Principal to set each linear factor = 0 and solve for *x*

Step 4: Verify result by graphing and finding the intersection point(s).

Example 2: Use an appropriate factoring method to solve each of the quadratic equations below. Show all of your work. Be sure to write your final solutions using proper notation. Verify your answer by graphing.

a) Solve by factoring: $-2x^2 = 8x$ Graph

b) Solve by factoring: $x^2 = 3x + 28$ Graph

Section 6.7: Simplifying Square Roots

The Product Property of Square Roots

If $a \neq 0$ and $b \neq 0$, then $\sqrt{a \cdot b} = \sqrt{a}\sqrt{b}$ and $\sqrt{a}\sqrt{b} = \sqrt{a \cdot b}$

Square Roots of Perfect Squares

$\sqrt{0} = 0$ $\sqrt{1} = 1$ $\sqrt{4} = 2$ $\sqrt{9} = 3$ $\sqrt{16} = 4$ $\sqrt{25} = 5$

$\sqrt{36} = 6$ $\sqrt{49} = 7$ $\sqrt{64} = 8$ $\sqrt{81} = 9$ $\sqrt{100} = 10$ $\sqrt{121} = 11$

Example 1: Simplify each of the following as much as possible. Leave answers in exact form.

Simplifying Square Roots of Composite Numbers:

$\sqrt{50} =$ $\sqrt{18} =$ $\sqrt{24} =$

Simplifying Square Roots of Prime Numbers:

$\sqrt{3} =$ $\sqrt{47} =$

Simplifying Square Roots with Fractions (I):

$\frac{\sqrt{50}}{2} =$ $\frac{\sqrt{18}}{6} =$ $\frac{\sqrt{24}}{4} =$

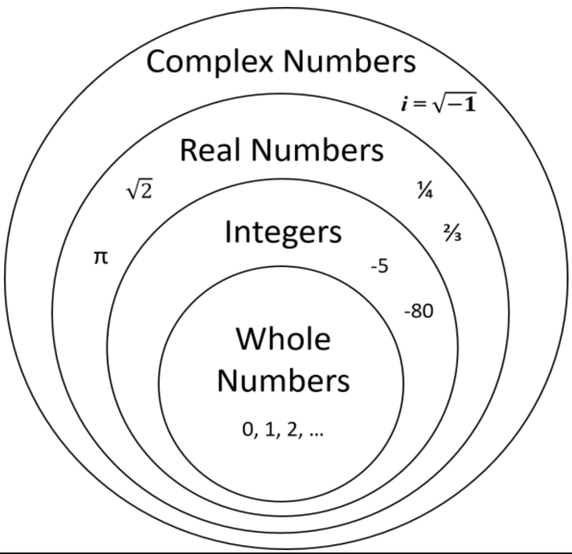
Simplifying Square Roots with Fractions (II):

$\frac{2+\sqrt{50}}{2} =$ $\frac{2-\sqrt{18}}{6} =$

$\frac{8+\sqrt{24}}{4} =$ $\frac{-4+\sqrt{16}}{12} =$

Section 6.8: Complex Numbers

Complex Numbers are numbers of the form *a + bi* such that *a* and *b* are real numbers $i = \sqrt{-1}$ $i^2 = -1$

Number Systems	Description and Examples
	**<u>Integers</u>** include positive and negative Whole Numbers. • Examples: -3, 0, 74 **<u>Real Numbers</u>** include Integers, Rational and Irrational Numbers. • Examples: -3, 0, 4.35, π, $\frac{2}{3}$ **<u>Complex Numbers</u>** include all Real Numbers and Integers. Complex numbers also include the imaginary unit $i=\sqrt{-1}$ • Examples: 3*i*, -5, 2 + 8*i*

Example 1: Simplify each of the following leaving your final result in *a + bi* form.

a) $\sqrt{-9} =$ b) $-\sqrt{-7}$ c) $\frac{3+\sqrt{-49}}{2}$

Example 2: Simplify each of the following leaving your final result in exact *a + bi* form.

a) $-4\sqrt{-25} =$ b) $\sqrt{32}$ c) $\frac{5-2\sqrt{-25}}{10}$

Example 3: Each of the following complex numbers is written in exact *a + bi* form. Change each of them to an approximate *a+bi* form rounded to the nearest thousandths place, if possible.

a. $3 + 4\sqrt{15}\, i$ b. $\frac{3}{7} + \left(-\frac{1}{7}\right) i$

Section 6.9: The Square Root Method

This method can be used to solve quadratic equations of the form: $ax^2 + c = 0$

The Square Root Property
The equation, $u^2 = d$ where $d > 0$, has exactly two solutions: $u = \sqrt{d}$ and $u = -\sqrt{d}$ These solutions can also be written as $u = \pm\sqrt{d}$.
The equation, $u^2 = d$ where $d < 0$, has exactly two solutions: $u = i\sqrt{\lvert d\rvert}$ and $u = -i\sqrt{\lvert d\rvert}$ These solutions can also be written as $u = \pm i\sqrt{\lvert d\rvert}$.
This solution process is also called *Extracting Square Roots.*

Example 1: Solve the equations below using the Square Root Property. Leave your solution(s) in exact form.

1. $x^2 - 9 = 0$

2. $x^2 + 64 = 0$

3. $2x^2 - 64 = 0$

4. $3x^2 + 7 = 0$

Example 2: Solve

$(3x - 5)^2 = 4$

$(x + 3)^2 = 72$

Section 6.10: Completing the Square

Solve a Quadratic Equation by Completing the Square

Step 1: Write the equation in the form $ax^2 + bx +$ _____ $= c +$ ______

Step 2: If $a \neq 1$, divide both sides of the equation by a.

Step 3: Add $\left(\frac{b}{2}\right)^2 = \left(\frac{1}{2}b\right)^2$ to both sides of the equation

Step 4: Factor the left side of the equation. *It should be a perfect square trinomial*. Write it as a binomial squared.

Step 5: Square root both sides of the equation to solve for x.

Example 1: $2x^2 + 7x + 5 = 0$

Section 6.11: The Quadratic Formula

The *Quadratic Formula* can be used to solve any quadratic equations written in standard form:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1: Solve the equation $x^2 – 3 = 3x$ using the Quadratic Formula. Leave your solution(s) in exact form and in approximate form rounded to the thousandths place.

Example 2: Solve the equation $-x^2 + 3x + 10 = 0$ using the Quadratic Formula. Verify your result by graphing and using the Intersection Method.

Example 3: Solve the equation $x^2 + 4x + 8 = 1$. Leave your results in the form of a complex number, a + bi.

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

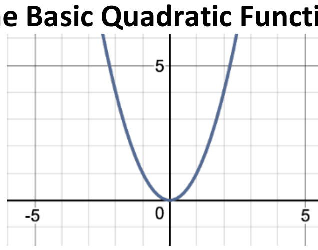
Unit 7: Quadratic Functions

Section 7.1: Characteristics of Quadratic Functions

A *quadratic function* is a function which can be written in the form $f(x) = ax^2 + bx + c$.

- $f(x) = ax^2 + bx + c$ has three distinct terms each with its own coefficient:
  - ax^2 is the first term and has coefficient a
  - bx is the second term and has coefficient b
  - c is the third term, called the constant term (We say that this term has coefficient c)
  - Note: If any term is missing, the coefficient of that term is 0
- *a, b,* and *c* can be any real numbers. Note that *a* cannot be 0.
- The graph of $f(x) = ax^2 + bx + c$ is called a *parabola*, is shaped like a U and opens either up or down.
- *a* determines which direction the parabola opens ($a > 0$ opens up, $a < 0$ opens down)
- *c* is the vertical intercept with coordinates $(0,c)$
- The domain of a quadratic function is $(-\infty, \infty)$

Graph of the Basic Quadratic Function: $f(x) = x^2$

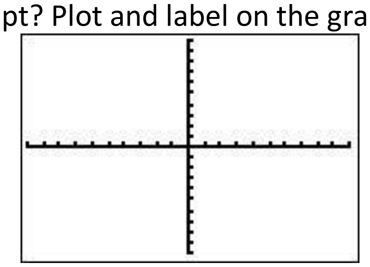


Example 1: Given the Quadratic Function $f(x) = x^2 - 2x + 3$

Identify the coefficients *a, b, c* ___________________________________

Which direction does the parabola open? Why? ______________________________

What is the vertical intercept? Plot and label on the graph. ____________________



Quadratic Functions: Vertex and Axis of Symmetry

Given a *quadratic function*, $f(x) = ax^2 + bx + c$

The ***vertex*** is the lowest or highest point of the associated parabola and is always written as an ordered pair $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

The ***axis of symmetry*** *equation* is the equation of the vertical line $x = -\frac{b}{2a}$ that passes through the vertex and divides the parabola in half.

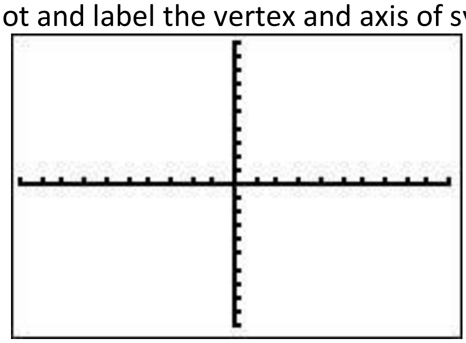
Example 2: Given the quadratic function $f(x) = x^2 - 2x + 3$.

Identify the coefficients *a, b, c* ___________________________________

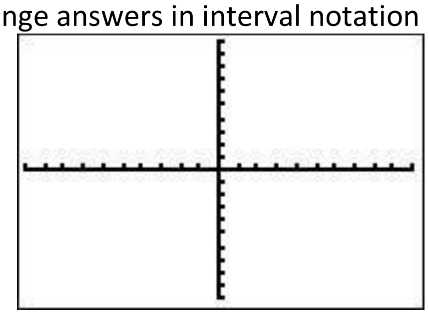
Determine the coordinates of the vertex.

Identify the axis of symmetry equation. ________________________

Graph the function. Plot and label the vertex and axis of symmetry on the graph.



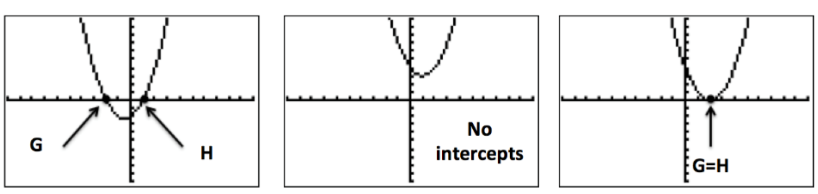
Example 3: Sketch the graph and find and label the vertex ordered pair for $f(x) = -2x^2 - 6$. Then, use the graph to help you determine the domain and range. Write your domain and range answers in interval notation and inequality notation.



Domain of $f(x)$

Range of $f(x)$

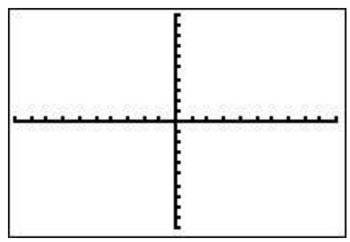
Section 7.2: Horizontal Intercepts of Quadratic Functions



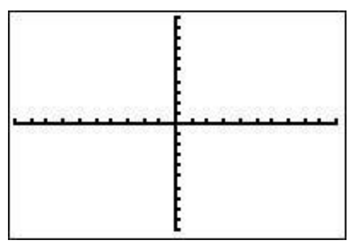
The quadratic function, $f(x) = ax^2 + bx + c$, will have *horizontal intercepts* if its parabola crosses the x-axis (i.e. if $f(x) = 0$). These intercepts are labeled as G and H on the graphs above.

Example 1: For each of the following functions, draw a sketch of the graph then use the Graphing /Intersection Method on your TI 83/84 calculator to identify the horizontal intercepts rounded to 2 decimal places. If these exist, label them on the graph. If there are no intercepts, indicate that as well.

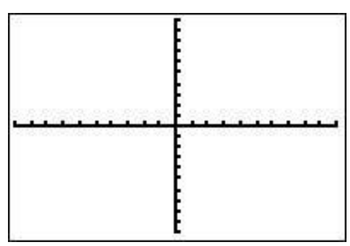
a. $f(x) = -2x^2 + 6x + 3$



b. $g(x) = x^2 - x + 2$



c. $h(x) = x^2 - 6x + 9$



Section 7.3: Graphing a Quadratic Function $f(x) = ax^2 + bx + c$

Example 1: Provide a detailed sketch of the quadratic function $g(x) = 3x^2 + 96x - 478$. Identify the characteristics of the graph and determine the domain and range.

- Vertical intercept: ___________
- Vertex: ____________
- Axis of Symmetry: _____________
- Horizontal intercept(s): _______________
- Domain: ______________
- Range: _____________

Graph

Example 2: Provide a detailed sketch of the quadratic function $f(x) = -4x^2 + 72x$. Identify the characteristics of the graph and determine the domain and range.

- Vertical intercept: ___________
- Vertex: ____________
- Axis of Symmetry: _____________
- Horizontal intercept(s) : ________________
- Domain: ______________
- Range: _____________

Graph

Example 3: Provide a detailed sketch of the quadratic function $h(x) = x^2 + 5$.
Identify the characteristics of the graph and determine the domain and range.

Graph

- Vertical intercept: ____________
- Vertex: _____________
- Axis of Symmetry: ______________
- Horizontal intercept(s) : _________________
- Domain: _______________
- Range: ______________

Example 4: Solve $x^2 – 10x + 1 = 4$. Plot and label the graphs and intersection points that are part of your solution process. Identify the final solutions clearly. Round to 2 decimal places.

Xmin = ___________
Xmax = ___________
Ymin = ___________
Ymax = ___________

Solution: _____________

Section 7.4: Quadratic Regression

Example 1: The table below shows the height, *H,* in feet, of a golf ball *t* seconds after being hit.

t	1	2	3	4	5
H(*t*)	81	131	148	130	87

a) Use the Quadratic Regression feature of your calculator to generate a mathematical model for this situation. Use function notation with appropriate variables. Round coefficients to two decimal places.

b) Use your model to predict the height of the golf ball at 5 seconds. Round your answer to two decimal places. How does this compare to the value in the data table?

c) Use your model to determine the maximum height of the golf ball. Round your answer to two decimal places.

d) Use your model to determine how long it will take the golf ball to hit the ground. Round your answer to two decimal places.

e) Use your model to determine the practical domain and practical range for this situation.

Practical Domain	Practical Range
______________________________	______________________________

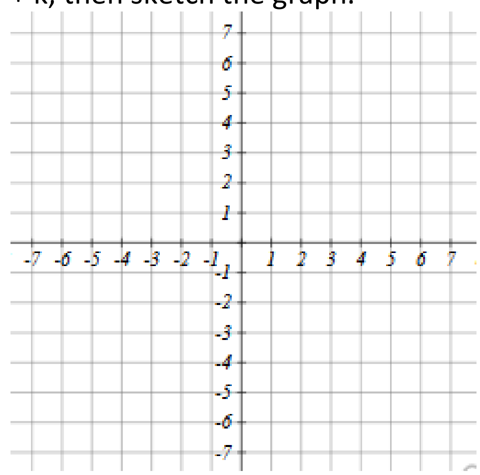
Section 7.5: Vertex Form of a Quadratic Function

Vertex Form of a Quadratic Function: $f(x) = a(x - h)^2 + k$

- Vertex: (h, k)
- Axis of Symmetry: x=h
- a > 0 opens Up
  a < 0 opens Down

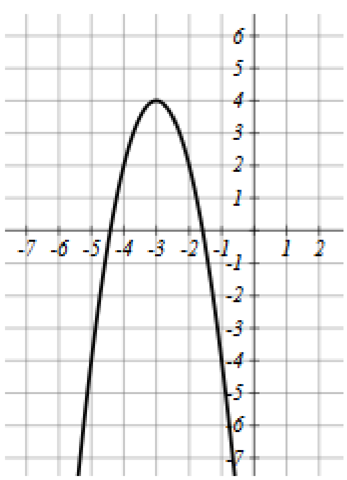
NOTE: Vertex Form is also called Standard Form.

Example 1: Write $y = x^2 + 2x - 4$ in the form $y = a(x - h)^2 + k$, then sketch the graph.



Example 2: Determine the equation of the quadratic function graphed below. Write your answer in both of the following forms: General Form: $f(x) = ax^2 + bx + c$

Vertex (Standard) Form: $f(x) = a(x - h)^2 + k$

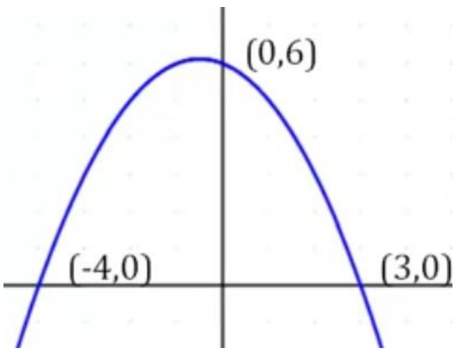


Section 7.6: Factored Form of a Quadratic Function

Factored Form for a Quadratic Function: $f(x) = a(x - r_1)(x - r_2)$

- **Horizontal intercepts:** $(r_1,0)$ and $(r_2,0)$
- a > 0 opens Up
  a < 0 opens down

Example 1: Determine the equation of the function from the graph.



Unit 8: Power and Polynomial Functions

Section 8.1: Negative and Rational Exponents

For any real numbers $a \neq 0$, $b \neq 0$, and m:

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m} \qquad a^{-m} = \frac{1}{a^{m}} \qquad \frac{1}{a^{-m}} = a^{m}$$

Example 1: Rewrite each of the following with only positive exponents. Variables represent nonzero quantities

a. $x^{-3} =$

b. $\frac{1}{x^{-3}} =$

c. $2^{-3} =$

d. $\left(\frac{4}{5}\right)^{-2} =$

e. $3x^{-4} =$

f. $(3x)^{-4} =$

Example 2: Simplify the following expressions. Variables represent nonzero quantities. Write your answer with only positive exponents.

a. $p^{-4} \cdot p^{2} \cdot p =$

b. $\frac{2}{3}a^{-5}b^{-3}c^{2} =$

c. $\frac{d^{-2}}{d^{-7}} =$

d. $\frac{4t^{-10}u}{6t^{-3}u^{-1}} =$

Basic Properties of Exponents

1. $a^m \cdot a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{m \cdot n}$
4. $a^{-m} = \frac{1}{a^m}$

Rational Exponents:

$$x^{\frac{1}{n}} = \sqrt[n]{x} \qquad x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m = \sqrt[n]{x^m}$$

Example 3: Rewrite each of the following as an equivalent expression with rational exponents.

a) $\sqrt[3]{x}$

b) $\sqrt[5]{r^2}$

c) $\sqrt{x^8}$, for $x \geq 0$

d) $\frac{1}{\sqrt[3]{b^5}}$

Example 4: Simplify.

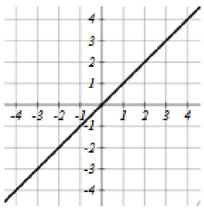
a) $8^{4/3} =$

b) $81^{3/4} =$

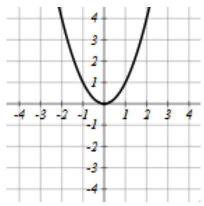
Section 8.2: Power Functions

A **Power Function** is a function that can be represented in the form: $f(x) = kx^p$ where p is any real number and $k \neq 0$. Some examples are shown below.

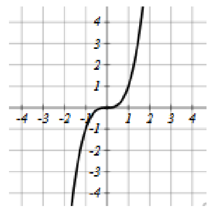
$f(x) = x$



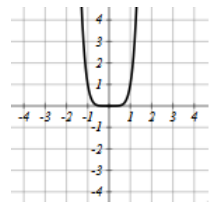
$f(x) = x^2$



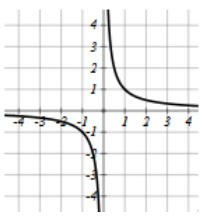
$f(x) = x^3$



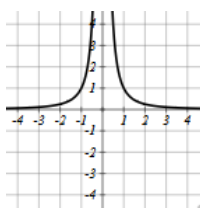
$f(x) = x^4$



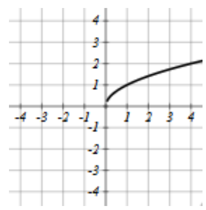
$f(x) = \frac{1}{x} = x^{-1}$



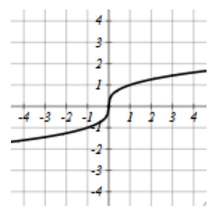
$f(x) = \frac{1}{x^2} = x^{-2}$



$f(x) = \sqrt{x} = x^{1/2}$



$f(x) = \sqrt[3]{x} = x^{1/3}$



Example 1: Which one of the following is not a power function?

○ $y = \frac{1}{x}$

○ $y = 3x^2$

○ $y = x^2 + 1$

○ $y = \sqrt{x}$

Section 8.3: Polynomial Functions

A Polynomial Function is a function of the form: $f(x)=a_nx^n + a_{n-1}x^{n-1} + ... + a_2x^2 + a_1x + a_0$

Key Information about Polynomials:

- **Leading Coefficient**: The coefficient of the leading term. For the polynomial above, the leading coefficient is a_n.
- **Constant Term**: A number with no variable factors. For the polynomial above, the constant term is a_0.
- **Degree**: The degree is the highest exponent in a polynomial. For the polynomial above, the degree is *n.*

Short-Run Behavior for a Polynomial Function: $f(x)=a_nx^n + a_{n-1}x^{n-1} + ... + a_2x^2 + a_1x + a_0$

- A polynomial of degree n has **at most n Horizontal Intercepts (real zeros, x-intercepts)**
- A polynomial of degree n has **at most n – 1 turning points**
- The **Vertical Intercept (y-intercept)** has coordinates: $(0,a_0)$

Example 1: Provide the information below given the polynomial function

$$f(x) = -2x^2 - 3x^6 + 7x + 4x^3 - 3$$

Descending Order:

Degree:

Leading Coefficient

Maximum number of real zeros:

Maximum number of turns:

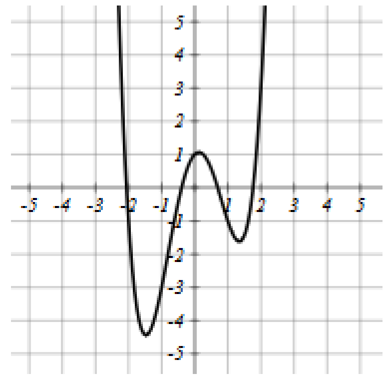
Example 2: Consider the polynomial functions shown below.

$f(x) = x^4 - 4x^2 + x + 1$

Degree: __________

Number of x-intercepts: __________

Number of turns: ___________

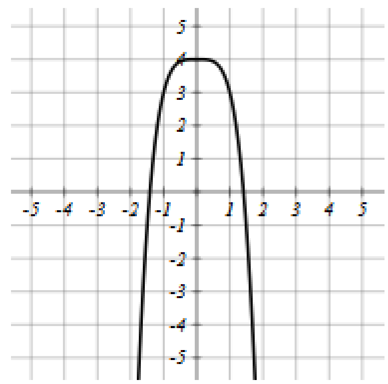


$f(x) = -4x^4 + 4$

Degree: __________

Number of x-intercepts: __________

Number of turns: ___________

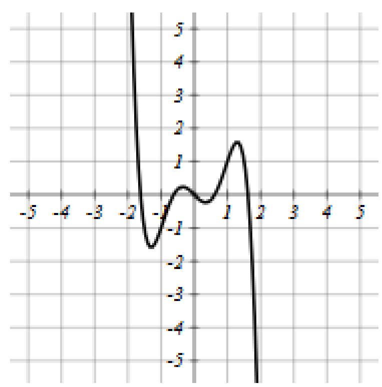


$f(x) = -x^5 + 3x^3 - x$

Degree: __________

Number of x-intercepts: __________

Number of turns: ___________

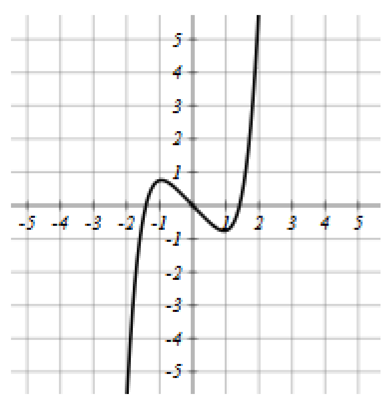


$f(x) = 0.25x^5 - x$

Degree: __________

Number of x-intercepts: __________

Number of turns: ___________



Section 8.4: End Behavior of Polynomial Function $f(x) = ax^n + \ldots$

End (Long-Run) Behavior describes the value of a function (y) as x approaches the left ($-\infty$) or the right (∞).

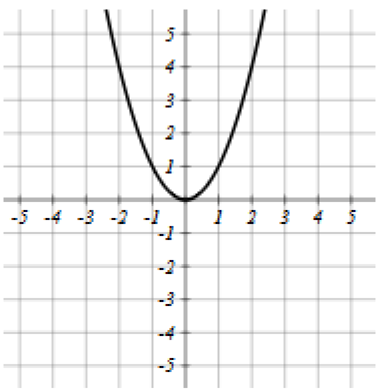
If the degree is EVEN ($n \geq 2$)

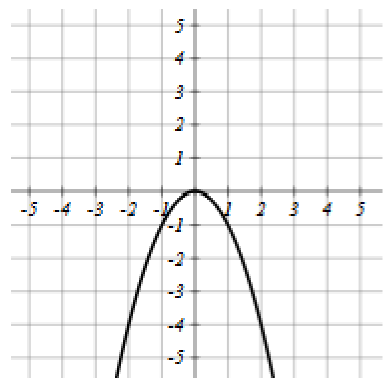
- $a > 0$ then as $x \to \pm\infty$, $f(x) \to \infty$ "Both tails point up"
- $a < 0$ then as $x \to \pm\infty$, $f(x) \to -\infty$ "Both tails point down"

If the degree is ODD

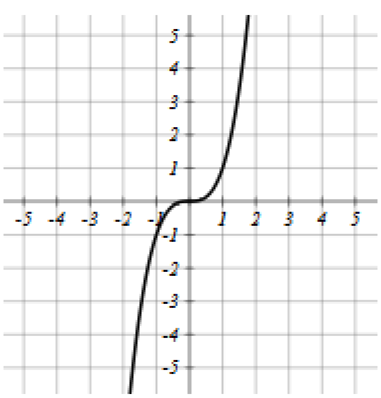
- $a > 0$ then as $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to -\infty$
- $a < 0$ then as $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to \infty$

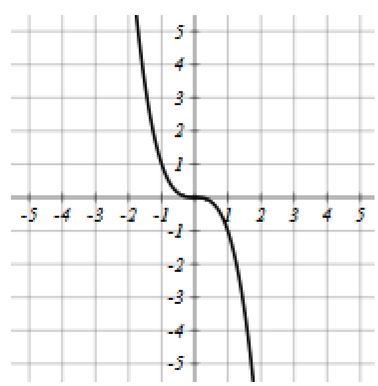
Example 1: EVEN DEGREE





Example 2: ODD DEGREE





Section 8.5: Intercepts

To determine the x-intercepts, set y=0 and solve for x.

To determine the y-intercept, set x=0 and solve for y.

Example 1: Determine the intercepts of the function: $f(x) = (x - 2)(x + 1)(x - 4)(x + 3)$

Example 2: Given $C(t) = 2t^4 - 10t^3 + 12t^2$, determine the vertical and horizontal intercepts.

Vertical Intercept (y-intercept):

Horizontal Intercepts (x-intercepts):

Section 8.6: Zeros and Multiplicity

Example 1: Consider the functions $f(x) = 3(x + 1)^4(x + 2)^2(5x + 1)^2(x – 2)^3$

State the zeros and multiplicity.

Zeros	Multiplicity

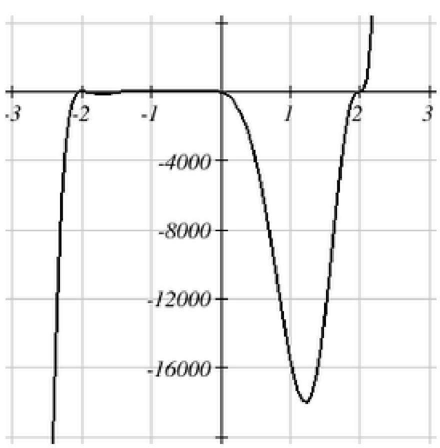
State the degree.

Determine the end behavior.

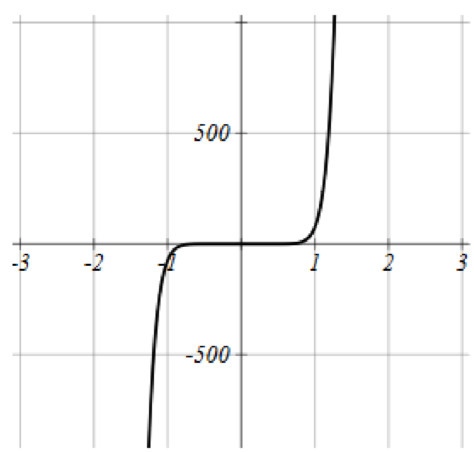
As $x \to -\infty$, $f(x) \to$ __________

As $x \to \infty$, $f(x) \to$ __________

$f(x) = 3(x + 1)^4(x + 2)^2(5x + 1)^2(x – 2)^3$



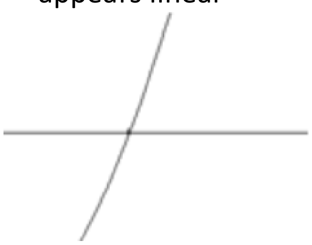
$g(x) = 75x^{11}$



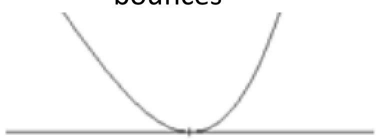
Section 8.7: Zeros and Multiplicity from a Graph

- If the graph *crosses* the horizontal axis at a zero, it is a zero with ODD multiplicity. If the graph appears *almost linear* at the zero, it is a single zero (multiplicity = 1)
- If the graph touches the horizontal axis ad bounces back at a zero, it is a zero with EVEN multiplicity
- The sum of the multiplicities must equal the degree of the polynomial.

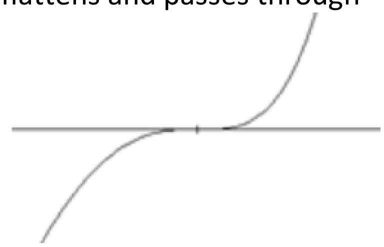
Multiplicity = 1
"appears linear"



Multiplicity is EVEN
"bounces"



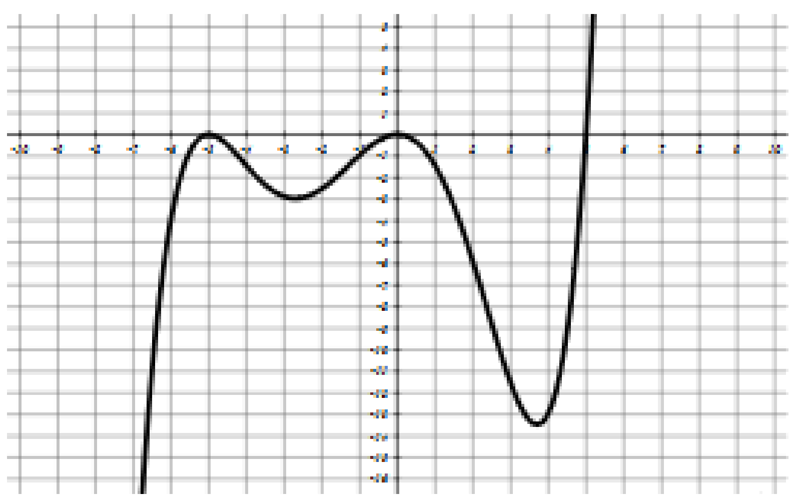
Multiplicity is ODD
"flattens and passes through"



NOTE: The higher the multiplicity, the "flatter" the graph will appear at that zero

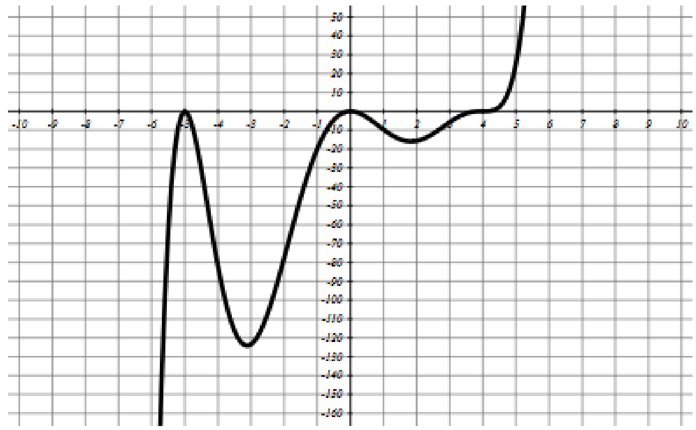
Example 1: Given the graph of a degree 5 polynomial, complete the table.

Root or Zero	Multiplicity
5	
	2



Example 2: Given the graph of a degree 7 polynomial, complete the table.

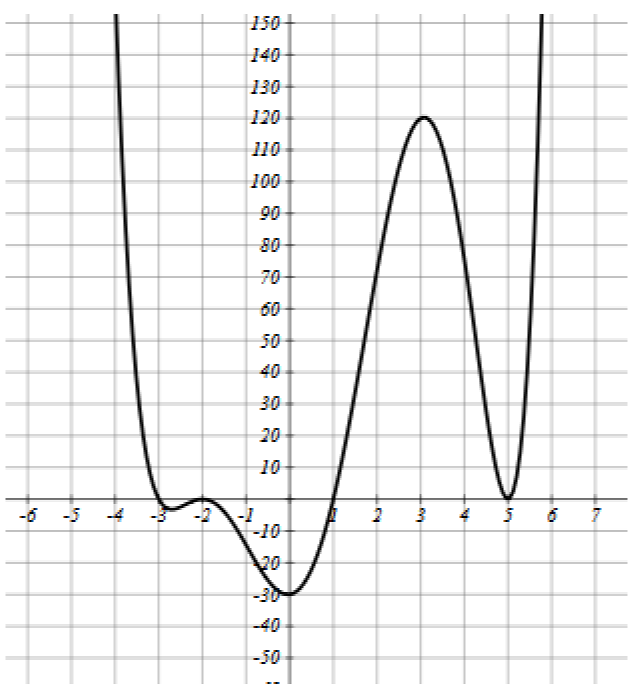
Root or Zero	Multiplicity
-5	
	2



Section 8.8: Writing the Equation for a Polynomial Function

Example 1: Find an equation of a polynomial function with real coefficients having zeros -4, -1, and 3.

Example 2: Find an equation for the graph of the degree 6 polynomial function Leave the function in factored form.

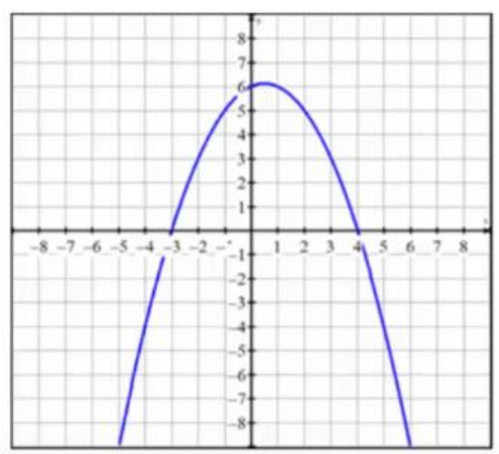
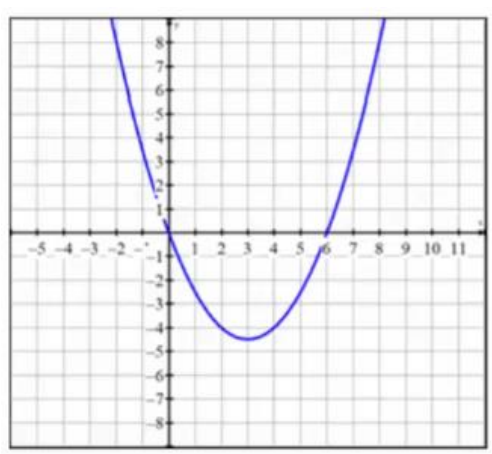


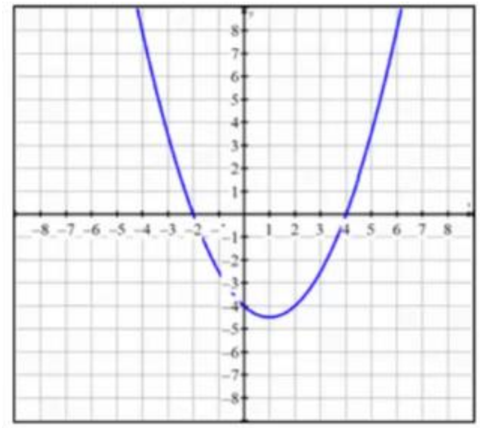
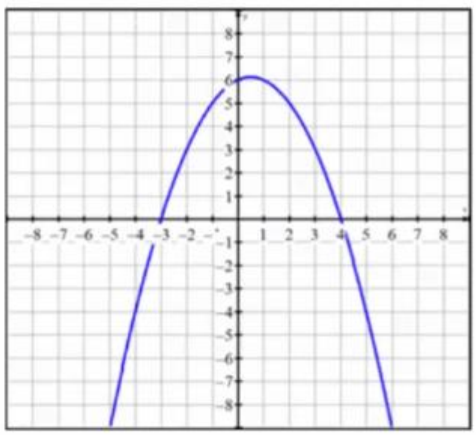
Section 8.9: Polynomial Inequalities

Inequalities from a Graph

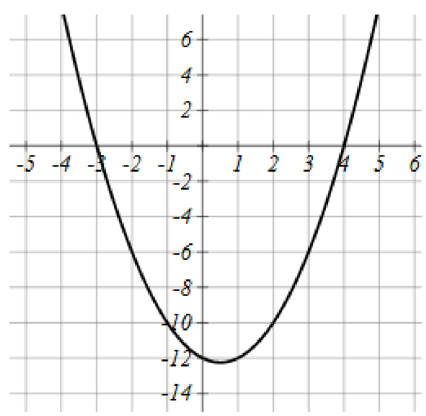
- $f(x)$ is **greater than 0** where the graph of $f(x)$ is **above** the x-axis.
- $f(x)$ is **equal to 0** where the graph of $f(x)$ **intersects** the x-axis.
- $f(x)$ is **less than 0** where the graph of $f(x)$ is **below** the x-axis.

Example 1: Solve the quadratic inequalities based upon the graphs provided. State the solution using inequalities, interval notation, and number lines.

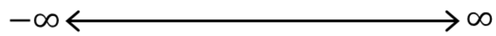
Graph	Inequality
	$f(x) > 0$ $-\infty \longleftrightarrow \infty$ Inequality Notation Interval Notation
	$f(x) \geq 0$ $-\infty \longleftrightarrow \infty$ Inequality Notation Interval Notation

	$f(x) \le 0$ $-\infty \longleftrightarrow \infty$ Inequality Notation Interval Notation
	$f(x) < 0$ $-\infty \longleftrightarrow \infty$ Inequality Notation Interval Notation

**Example 2**: Solve the quadratic inequality, $x^2 - x - 12 \le 0$.



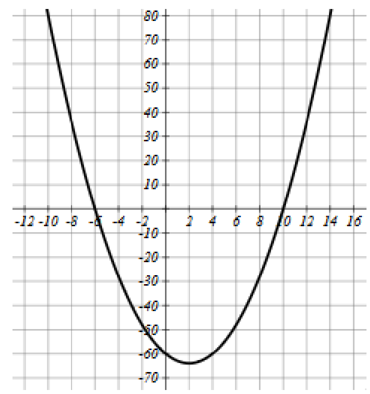
Example 3: Solve the quadratic inequality, $x^2 \geq 4x + 60$.



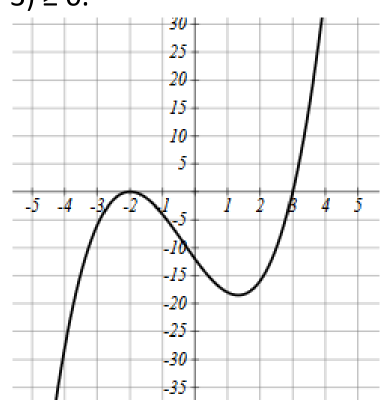
Inquality

Interval Notation

Graph: $x^2 \geq 4x + 60 \rightarrow x^2 - 4x - 60 \geq 0$



Example 4: Solve the factored polynomial inequality $(x + 2)^2(x - 3) \geq 0$.



Unit 9: Rational Functions

Section 9.1: Fractions Review

Writing Fractions in Simplest Form.

Example 1: Write the following fractions in simplest form.

$\frac{4}{16}$ $\frac{28}{54}$ $\frac{360}{495}$

Addition and Subtraction of Fractions

Example 2: Perform the indicated operations

a. $\frac{1}{2}+\frac{1}{3}$

b. $\frac{11}{15}-\frac{5}{12}$

c. $4\frac{3}{5}-1\frac{5}{6}$

d. $2-\frac{8}{5}$

Multiplication of Fractions

Example 3: Multiply. Write your answers in simplest form

a. $\frac{2}{3}\times\frac{3}{4}$

b. $\frac{12}{25}\times\frac{35}{48}$

c. $\frac{7}{8}\times 5$

d. $3\frac{1}{5}\times 1\frac{1}{9}$

Division of Fractions

Example 4: Divide. Write your answers in simplest form.

a. $\frac{1}{2} \div \frac{3}{5}$

b. $8 \div \frac{4}{5}$

Order of Operations with Fractions

Example 5: Perform the indicated operations. $\frac{1}{2} + \frac{3}{2} \times \frac{2}{5}$

Section 9.2: Rational Expressions

Definition: Let u and v be polynomials. The algebraic expression $\frac{u}{v}$ is a rational expression. The domain of this rational expression is the set of all real numbers for which v ≠ 0.

Examples: $\frac{2}{3}$, $\frac{m}{n}$, $\frac{2x+3}{x-8}$, $\frac{x^2-4x-12}{2x^3-19}$

Example 1: Simplify the following rational expressions. Begin by determining the domain.

a) $\frac{2x-4}{x^2-x-2}$

b) $\frac{x^2-9}{x^2-3x}$

Example 2: Subtract $\frac{x+1}{x} - \frac{x-2}{x-3}$

Example 3: Multiply $\frac{1}{y-6} \times \frac{y^2+4y-12}{y+6}$

Example 4: Divide $\frac{\frac{1-x}{3x}}{\frac{x-1}{x+4}} = \frac{1-x}{3x} \div \frac{x-1}{x+4}$.

Section 9.3: Solving Rational Equations

Guidelines for solving Rational Equations:

1. Factor all of the denominators
2. Multiply each side of the equation by the least common denominator (LCD)
3. Solve the resulting equation
4. Check your solutions in the original equation. NOTE: You must *exclude* the values that make the denominator equal to zero.

Example 1: $\frac{2}{3} - \frac{5}{6} = \frac{1}{t}$

Example 2: $\frac{x+2}{x-6} = \frac{x-1}{x+2}$

Example 3: $\frac{x}{x+3} - \frac{x}{x-2} = \frac{10}{x^2+x-6}$

Example 4: $\frac{4y}{y+2} - \frac{3y}{y-1} = \frac{y^2-8y-4}{y^2+y-2}$

Example 5: $\frac{2}{5x} - 3 = \frac{4}{x}$

Example 6: $2x - \frac{16}{x} = 4$

Section 9.4: Introduction to Rational Functions

A *rational function* is a function of the form $f(x) = \frac{p(x)}{q(x)}$ where *p*(*x*) and *q*(*x*) are polynomials and *q*(*x*) does not equal zero.

Example 1: Determine the domain of the following rational functions.

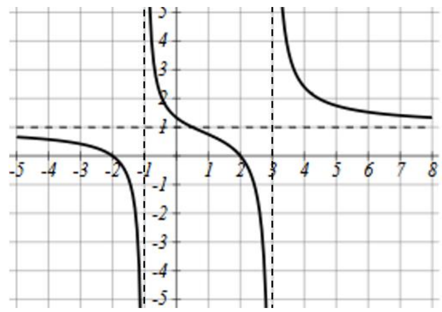
$f(x) = \frac{4}{2x-5}$

$g(x) = \frac{x}{2x^2+3x}$

$h(x) = \frac{x^2-4x}{x^2+4x-21}$

Section 9.5: Characteristics of Rational Functions

Example 1: Consider the rational function $f(x) = \frac{x^2-4}{x^2-2x-3} = \frac{(x+2)(x-2)}{(x+1)(x-3)}$



Vertical Asymptotes:

- The line x = a is a vertical asymptote of the graph of the function f(x) if a is a zero of the denominator and does not come from a common factor with the numerator.
- If x = a is a vertical asymptote, as x →a, f(x) → ±∞.

Holes:

- Zeros of the denominator that are also zeros of the numerator result in a hole in the graph.

Example 2: $f(x) = \frac{x^2-9}{x^2-5x+6}$

Horizontal Asymptotes:

- Horizontal Asymptotes are horizontal lines that the graph of the function approaches as x → ±∞

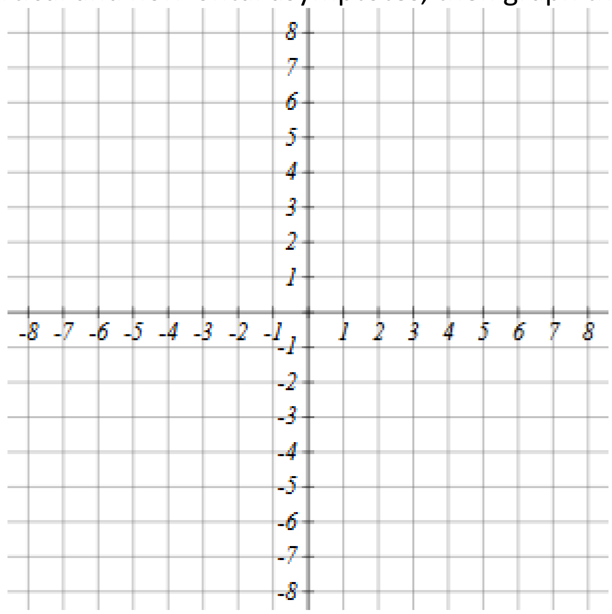
The race to infinity:

- If the degree of the numerator is higher than the degree of the denominator, there is no horizontal asymptote.
- If the degree of the numerator and denominator are equal, the horizontal asymptote is the ratio of the leading coefficients.
- If the degree of the denominator is higher than the degree of the numerator, the horizontal asymptote is y = 0.

Example 3: $f(x) = \frac{2x^3}{x+1}$ $f(x) = \frac{2x}{x+1}$ $f(x) = \frac{2}{x+1}$

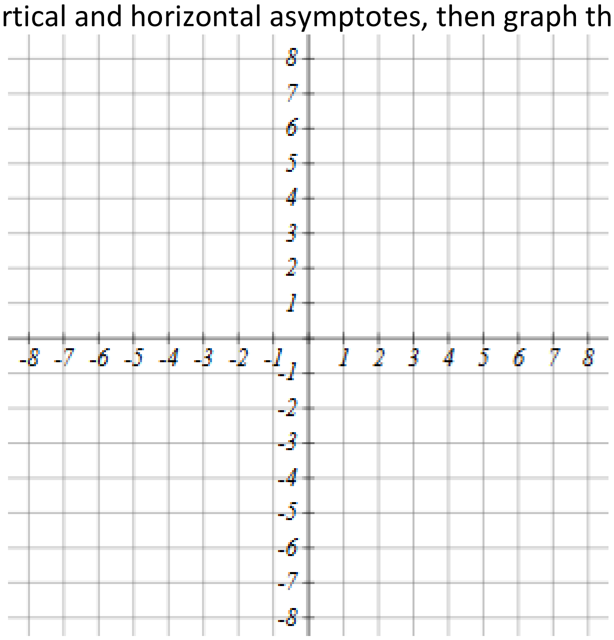
Example 4: Determine the vertical and horizontal asymptotes, then graph the function.

$$f(x) = \frac{4}{x+1}$$



Example 5: Determine the vertical and horizontal asymptotes, then graph the function.

$$f(x) = \frac{x^2}{x^2 - 2x}$$



Example 6: Determine the domain, location of holes, and equations of vertical and horizontal asymptotes.

$f(x) = \frac{(x-7)(x+5)}{(x+6)(x-4)(x-1)}$

Domain: ______________________

Holes: ________________________

Vertical Asymptotes: ___________

Horizontal Asymptote: ___________

Example 7 Consider the function $f(x) = \frac{15x-12}{3x+4}$

a) What is the domain?

b) Give the equation of the vertical asymptote for $g(x)$.

c) Give the equation of the horizontal asymptote for $g(x)$.

d) What is the vertical intercept? Show your work. Write as an ordered pair.

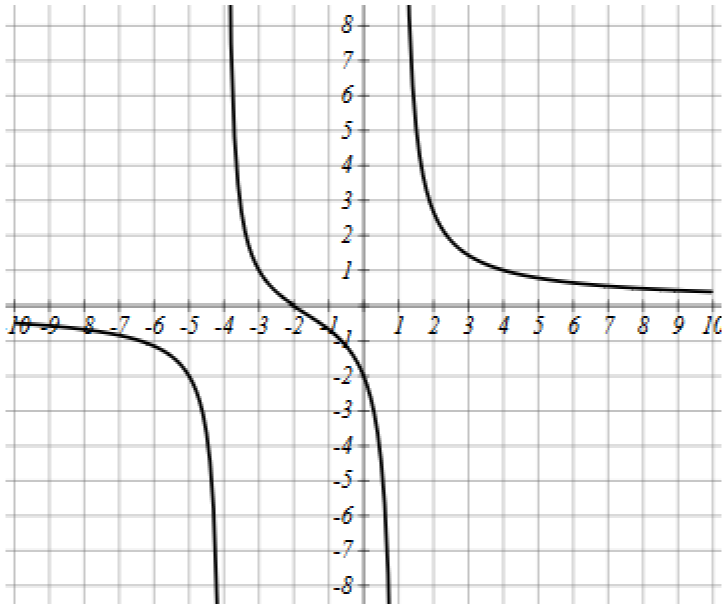
e) What is the horizontal intercept? Show your work. Write as an ordered pair.

f) Determine $g(12)$. Show your work.

g) For what value of x is $g(x) = 3$? Show your work.

Section 9.6: Writing Rational Functions

Example 1: Write and equation for the rational function.



Section 9.7: Applications of Rational Functions

Example 1: You and your family are driving to San Diego on a road trip. From Phoenix, the trip is 354.5 miles according to Google. Answer the following questions based upon this situation.

a) Use the relationship, Distance = Rate times Time or $d = rT$, to write a rational function, T(r), that has the average rate of travel, r (in mph), as its input and the time of travel (in hours) as its output. The distance will be constant at 354.5 miles.

b) Provide a rough but accurate sketch of the graph in the space below. Label your horizontal and vertical axes. You only need to graph the first quadrant information. Indicate the graphing window you chose.

Xmin = ______

Xmax = ______

Ymin = ______

Ymax = ______

c) If you average 60 mph, how long will the trip take? Write your answer in a complete sentence.

d) If the trip took 10 hours, what was your average rate of travel? Write your answer in a complete sentence.

e) Determine the horizontal asymptote for T(r) and explain its meaning in this situation.

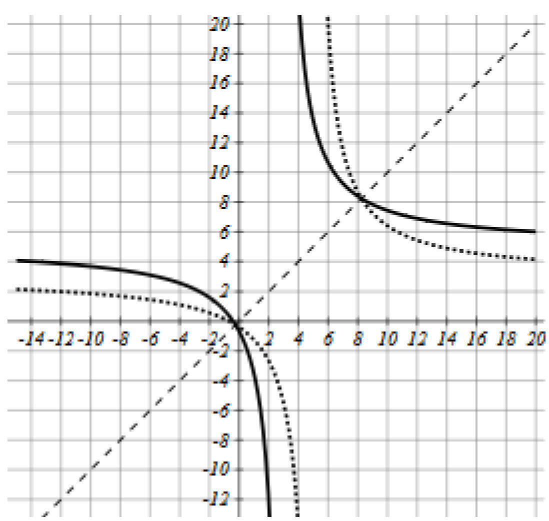
f) Determine the vertical asymptote for T(r) and explain its meaning in this situation.

Example 2: Ria can paint a room in 4 hours. Destiny can paint the same room in 6 hours. How long does it take for both Hannah and Destiny to paint the room it they are working together?

Example 3: One inlet pipe can fill an empty pool in 8 hours, and a drain can empty the pool in 12 hours. How long will it take the pipe to fill the pool if the drain is left open?

Section 9.8: Find the Inverse of a Rational Function

Example 1: Let $f(x) = \frac{5x+2}{x-3}$. Find f^{-1}



Section 9.9: Average Rate of Change and Difference Quotient

Example 1: Find the average rate of change of $f(x) = \frac{5x+2}{x-3}$ on the interval [3, 3+h]

$$\text{Average Rate of Change} = \frac{\text{Change in OUTPUT}}{\text{Change in INPUT}} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 2: Find and simplify the difference quotient for the function $f(x) = \frac{2}{x}$

$$\text{Difference Quotient} = \frac{f(x+h) - f(x)}{h}$$

Unit 10: Roots and Radical Functions

Section 10.1 – Adding and Subtracting Radical Expressions

To add and subtract radicals, you must have what are called "like" radicals.

Example 1: Add or subtract the radical expressions as indicated and simplify.

a) $5\sqrt{2} - 3\sqrt{2}$

Start by factoring $\sqrt{2}$ from each term and then simplify to obtain the final result

b) $5\sqrt{3} + 2\sqrt{2} - 4\sqrt{3}$

The terms with $\sqrt{3}$ can be combined because they are like terms. The $2\sqrt{2}$ term must stand on its own

c) $-2\sqrt{8} + 2\sqrt{2}$

Start by simplifying $\sqrt{8}$ in the first term. Then factor $\sqrt{2}$ from each term and simplify to obtain the final result

Example 2: Add or subtract the radical expressions as indicated and simplify.

a) $4\sqrt{5} + \sqrt{5}$

b) $4\sqrt{7} - 3\sqrt{11} - 4\sqrt{7} + 5$

c) $4\sqrt{5} - 7\sqrt{25}$

Section 10.2 – Multiplying Radical Expressions

The Product Property of Square Roots:

For $a \neq 0$ and $b \neq 0$, $\sqrt{a \cdot b} = \sqrt{a}\sqrt{b}$ and $\sqrt{a}\sqrt{b} = \sqrt{a \cdot b}$

Example 1: Multiply the radical expressions as indicated and simplify.

a) $\sqrt{3}\sqrt{3} = \sqrt{3 \cdot 3} = \sqrt{9} = 3$

Use the Product Property to combine the radicals and then multiply the 3's to get $\sqrt{9}$ which simplifies to 3.

b) $\sqrt{5}\sqrt{10} = \sqrt{5 \cdot 10} = \sqrt{50} = 5\sqrt{2}$

Use the Product Property to combine the radicals and then multiply to get $\sqrt{50}$. Note that we also use the Product Property when we write $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$

c) $\sqrt{3}\left(2 - \sqrt{5}\right)$

Multiply $\sqrt{3}$ times each term in ( ). Then use the Product Property to combine $\sqrt{3}\sqrt{5} = \sqrt{15}$ and obtain the final result.

d) $\left(1 + \sqrt{3}\right)^2$

Use the FOIL method to approach this problem multiplying each term in the first ( ) by each term in the 2nd ( ). Then simplify by combining like radicals and obtain the final result

Example 2: Multiply the radical expressions as indicated and simplify.

a) $5\sqrt{7}\left(5 - 2\sqrt{3}\right)$

b) $\left(-3 + 2\sqrt{5}\right)^2$

Section 10.3 – Rationalizing the Denominator

If a radical division expression has radicals in the denominator, we need to continue simplifying. The method we use to remove radicals from the denominator is called *rationalizing the denominator*.

Example 1: Divide the radical expressions as indicated and simplify leaving the denominator free of radicals.

a) $\frac{1}{\sqrt{5}}$

b) $\frac{3}{\sqrt{20}}$

Section 10.4 – Rational Exponents

Basic Properties of Exponents Examples

1. $a^m \cdot a^n = a^{m+n}$

2. $\dfrac{a^m}{a^n} = a^{m-n}$

3. $(a^m)^n = a^{m \cdot n}$

4. $a^{-m} = \dfrac{1}{a^m}$

$$x^{1/n} = \sqrt[n]{x} \qquad x^{m/n} = \left(\sqrt[n]{x}\right)^m = \sqrt[n]{x^m}$$

Example 1: Rewrite each of the following as an equivalent expression with rational exponents.

a) $\sqrt[3]{x}$

b) $\sqrt[5]{r^2}$

c) $\sqrt{x^8}$, for $x \geq 0$

d) $\dfrac{1}{\sqrt[3]{b^5}}$

Example 2: Simplify.

a) $8^{4/3} =$

b) $81^{3/4} =$

Example 3: Use your calculator to compute each of the following in the real number system. Round to two decimal places as needed.

a) $\sqrt{49}$

b) $8^{1/3}$

c) $\sqrt{-49}$

d) $\sqrt[3]{-8}$

e) $-25^{3/2}$

f) $(-25)^{3/2}$

g) $\sqrt[7]{49}$

h) $\sqrt[4]{12^3}$

Section 10.5 – Key Characteristics of Square Root Functions

Example 1: For each of the functions below, determine the domain, horizontal intercept, and the vertical intercept (if it exists). Then sketch an accurate graph of each. Use interval notation for the domain. Round to two decimal places as needed.

a) $f(x) = \sqrt{12 - 4x}$

Domain:

Horizontal Intercept:

Vertical Intercept:

b) $g(x) = \sqrt{2x - 5}$

Domain:

Horizontal Intercept:

Vertical Intercept:

Section 10.6 – Key Characteristics of Cube Root Functions

Example 1: For each of the functions below, determine the domain, horizontal intercept, and the vertical intercept (if it exists). Then sketch an accurate graph of each. Use interval notation for the domain. Round to two decimal places as needed.

a) $f(x) = \sqrt[3]{27 - 15x}$

Domain:

Horizontal Intercept:

Vertical Intercept:

b) $g(x) = \sqrt[3]{2x - 10}$

Domain:

Horizontal Intercept:

Vertical Intercept:

Section 10.7 – Solve Radical Equations Algebraically

Solving Radical Equations Algebraically

1. Isolate the radical part of the equation on one side and anything else on the other side.
2. Sometimes you will have radicals on both sides.
3. Raise both sides of the equation to a power that will "undo" the radical (2^{nd} power to eliminate square root, 3^{rd} power to eliminate cube root).
4. Solve for the variable.
5. Check your answer! Not all solutions obtained will check properly in your equation.

Example 1: Solve the equations **algebraically**. Write your answers in exact form, then give the decimal approximation rounded to the nearest hundredth. Be sure to check your final result!

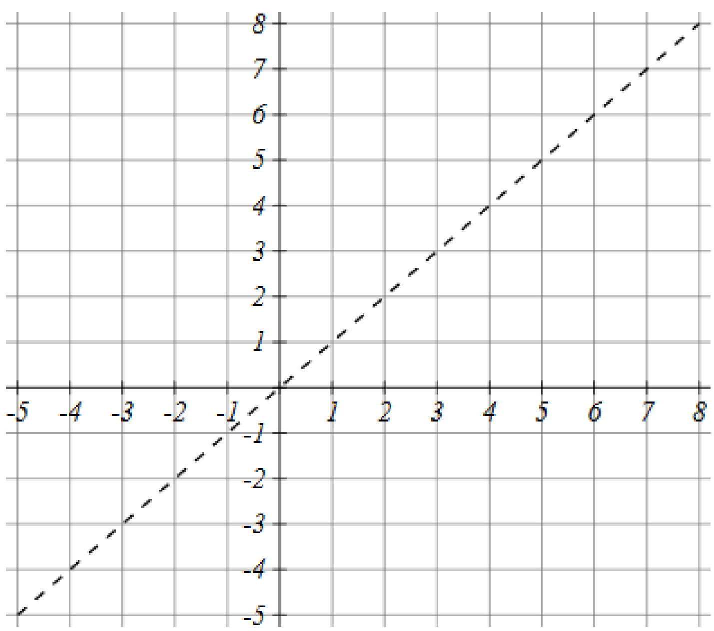
a) $\sqrt[3]{2x-1}=5$

b) $41+5\sqrt{2x-4}=11$

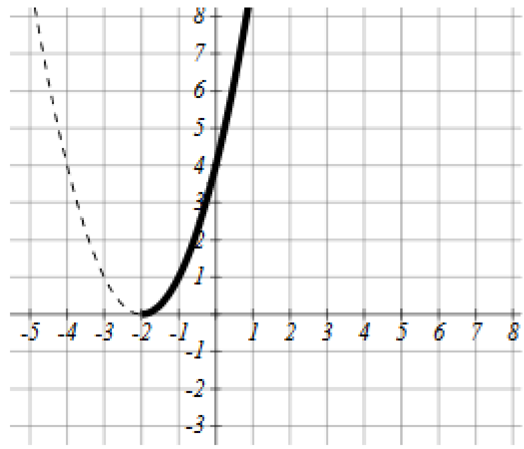
Example 2: Solve the equation algebraically ***and*** graphically: $1+\sqrt{7-x}=x$.
Be sure to check your final result!

Section 10.8 – Inverse Functions

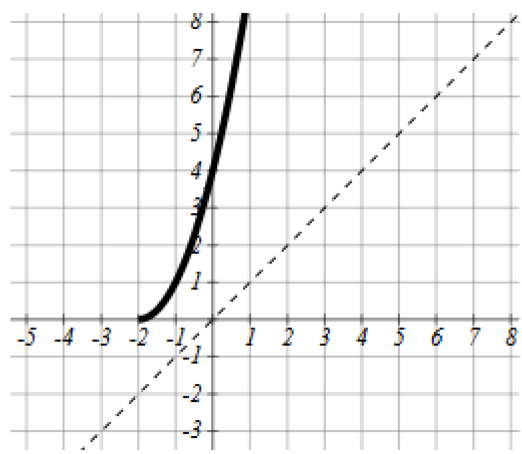
Example 1: Let $f(x) = \sqrt{2x-1} - 3$. Determine the domain and range, then find $f^{-1}(x)$.



Example 2: Given $f(x) = (x+2)^2$. Determine the domain so $f(x)$ is increasing and one-to-one. Also give the range. Use interval notation.



Find $f^{-1}(x)$. Give the domain and range.

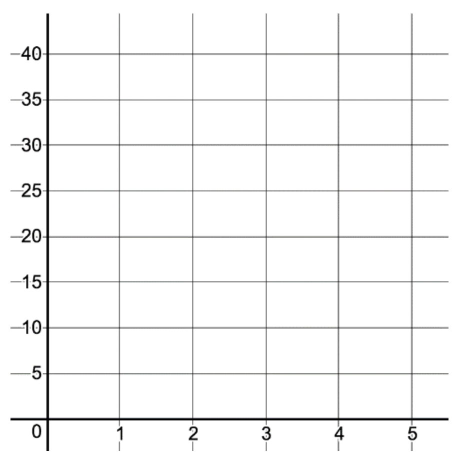


Unit 11: Exponential Functions

Section 11.1: Introduction to Exponential Functions

Example 1: Complete the input/output table for each of the functions below then graph each function on the grid provided.

x	$f(x) = 2x$	$g(x) = 2^x$
0		
1		
2		
3		
4		
5		



What do you notice about the outputs in the table above and about the graphs for each function?

Example 2: After graduating from college, Tristan receives two different job offers. Both pay a starting salary of $58,000 per year, but one job promises a $2,000 raise per year, while the other guarantees a 5% raise each year.

Complete the tables below to determine what Tristan's salary will be after *t* years.
Round your answers to the nearest dollar.

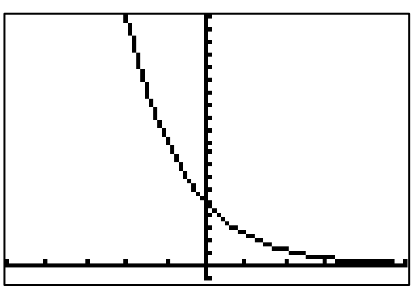
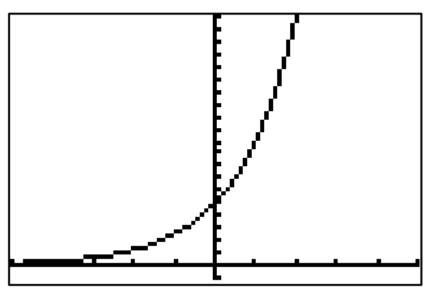
Year, t	Salary with $2000 raise each year
1	
2	
3	
5	
10	
20	

Year, t	Salary with 5% raise each year
1	
2	
3	
5	
10	
20	

Section 11.2: Characteristics of Exponential Functions

Exponential Functions are of the form $f(x) = ab^x$ where:

- The input, x, is located in the *exponent*.
- a is the **initial value**. This can be found by computing $f(0) = ab^0 = a(1) = a$.
- b is the **base** ($b > 0$ and $b \neq 1$); also called the *growth factor* or *decay factor*.
    - If $b > 1$, the function is an *exponential growth function*, and the graph increases from left to right.
    - If $0 < b < 1$, the function is an *exponential decay function*, and the graph decreases from left to right.
- Vertical intercept: $(0, a)$
- Horizontal Intercept: DNE
- Domain: All Real Numbers or $(-\infty, \infty)$
- Range: $f(x) > 0$ or $(0, \infty)$
- Horizontal Asymptote: $y = 0$

Exponential Decay	Exponential Growth
	
$0 < b < 1$ **Vertical Intercept: $(0, a)$**	**$b > 1$** **Vertical Intercept: $(0, a)$**

Example 1: Complete the table below for each of the functions. Draw your graph by hand first, then confirm using your calculator and the window: X[-10..10], Y[0..500].

	$f(x) = 150(1.25)^x$	$g(x) = 150(0.75)^x$
Graph		
Initial Value		
Base		
Domain		
Range		
Horizontal Intercept		
Vertical Intercept		
Horizontal Asymptote		
Increasing or Decreasing		

Section 11.3: Growth and Decay Rates

An exponential function $f(x) = ab^x$ grows or decays at a constant percent rate, r.

r = growth/decay rate in decimal form

Growth Factor: $b = 1 + r$ ------------------ Growth Rate: $r = b - 1$

Decay Factor: $b = 1 - r$ ------------------ Decay Rate: $r = 1 - b$

Example 1: Complete the following table.

Exponential Function	Growth or Decay?	Initial Value	Growth/Decay Factor	Growth/Decay Rate, r (as a decimal)	Growth/Decay Rate, r (as a %)
$f(x) = 812(0.71)^x$					
$g(t) = 64.5(1.32)^t$					
	Growth	8.24			0.5%
	Decay	150			20%

Example 2: The equation $V = 355000(1.06)^t$ represents the value (in dollars) of a house t years after its purchase. Use this equation to complete the statements below.

The value of the house is ________________ at a rate of _________________________________.

The purchase price of the house was ____.

The value of the house after 5 years is ____.

Round to the nearest dollar.

Section 11.4: Linear and Exponential Models

Example 1: When it first opened, there were 700 students enrolled in a new charter school. Using function notation, write a formula for the function $N(t)$ which represents the number of students enrolled in this charter school after t years, assuming that the enrollment

a) Increases by 20 students per year.

b**)** Increases by 20% per year.

c) Decreases by 20 students each year.

d**)** Decreases by 20% per year.

Section 11.5: Solving Exponential Equations

Example 1: Solve the following equations by graphing. Include a rough but accurate sketch of the graphs and intersection point. Mark and label the intersection. Round answers to two decimal places.

a) Solve $50 = 25(1.15)^x$ Solution: x = _______________

Xmin = __________

Xmax = __________

Ymin = __________

Ymax = __________

b) Solve $250 = 25(1.15)^x$ Solution: x = _______________

Xmin = __________

Xmax = __________

Ymin = __________

Ymax = __________

c) Solve $5 = 10(0.86)^t$ Solution: t = _______________

Xmin = __________

Xmax = __________

Ymin = __________

Ymax = __________

Example 2: In 2001, the population of a particular city was 22,395 with an identified growth rate of 6.2% per year. Assume that this growth rate is fairly consistent from year to year.

a) Write the exponential growth model for this situation.

Initial Population: __________

Given growth rate as a decimal: ___________

Growth factor: _______________

Write the model: $P(t) = a(b)^t$ ____________________

b) What is the approximate population of the city in 2006? Be sure to round to the nearest whole person.

c) Estimate the number of years (to the nearest whole year) that it will take for the population to double. In what actual year will this take place? Be sure to set up and clearly identify the Doubling Equation. Then, draw a sketch of the graph you obtain when using the Graphing/Intersection Method to solve. Round to the nearest whole year.

Doubling Equation: ____________________________________

Example 3: The 2000 U.S. Census reported the population of Tulsa, Oklahoma to be 382,872. Since the 2000 Census, Tulsa's population has been decreasing at approximately 2.6% per year.

a) Write an EXPONENTIAL DECAY MODEL, $P(t)$, that predicts the population of Tulsa, OK at any time t.

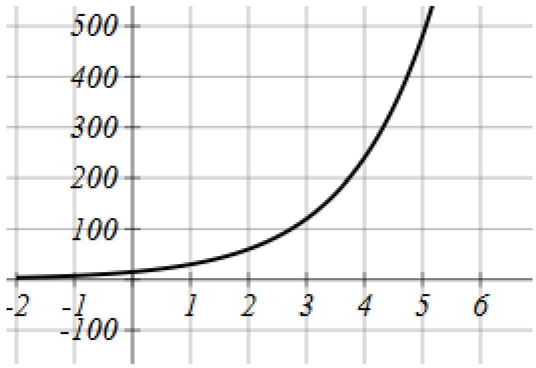
b) Use the function you wrote for $P(t)$ to predict the population of Tulsa, OK in 2013.

c) In how many years will the population of Tulsa decrease to 300,000 people (round to the nearest whole year)?

d) In how many years will the population of Tulsa decrease to HALF of the initial (2000) population? Round to the nearest whole year.

Section 11.6: Determine an Exponential Function Given Two Points

Example 1: Determine the formula for the exponential function that passes through the points (0,15) and (5,480). Write your answer in the form $f(x) = ab^x$



Section 11.7: Exponential Regression

Example 1:

t	0	1	2	4	6	9	12	16
P(t)	125	75	50	32	22	16	10	5.7

a) Use your calculator and the steps for exponential regression to generate an exponential function model, P(t) = ab^t, for the table above. Round a to 1 decimal place and b to three decimal places.

b) Use your graphing calculator to generate a scatterplot of the data and the graph of the regression equation on the same screen. You must use an appropriate viewing window. In the space below, draw what you see on your calculator screen, and write down the viewing window you used.

Xmin = ____________

Xmax = ____________

Ymin = ____________

Ymax = ___________

c) What is the rate of decay (as a %) for this function? __________________________

d) Determine P(20). Show your work. Write the corresponding ordered pair. Round to two decimal places.

e) Using your equation from part a), determine t when P(t) = 28. Show your work. Write the corresponding ordered pair. Round to two decimal places.

Section 11.8: Exponential Functions with Base *e*: $f(x) = ae^{kx}$

The base $e \approx 2.718$ is an irrational constant, often referred to as Euler's Number

For these exponential functions:

- k is the continuous growth rate, as a decimal.
  If $k > 0$, the function represents exponential growth with a continuous growth rate of k
  If $k < 0$, the function represents exponential decay with a continuous decay rate of k
- *a* is the initial value or starting amount
- The vertical intercept is $(0,a)$

Example 1: The number of bacteria in a culture is n(t) after t hours. $n(t) = 500e^{0.15t}$

What is the growth rate as a percent?

What is the initial population?

How many bacteria are present after 12 hours?

Example 2: Fill in the table for each function:

Continuous: $y = ae^{kt}$

Function	k	Growth or Decay? Why?	% Change
$y = 100e^{0.3t}$			
$y = 100e^{-0.3t}$			

Non-Continuous: $y = ab^t$

Function	b	Growth or Decay? Why?	% Change
$y = 100(1.3)^t$			
$y = 100(0.7)^t$			

Example 3: Write the formulas for each situation described below.

Write a formula for a quantity that starts at 100 and grows by 7% per year.

Write a formula for a quantity that starts at 100 and decreases by 7% per year.

Write a formula for a quantity that starts at 100 and grows at a continuous rate of 7% per year.

Write a formula for a quantity that starts at 100 and decreases at a continuous rate of 7% per year.

Example 4: Certain medical tests involve injecting radioactive materials into the body. One medical test injects 0.5mL of Technetium-99m that decays at a continuous rate of 11.55% per hour. How much Technetium-99m is left in the body after 24 hours?

Example 5: A population of bacteria is growing according to the equation $P(t)=1000e^{0.12t}$, where P(t) is the population and t is the time in hours. Estimate when the population will reach 5000. Round to the tenths place.

Section 11.9: Growth/Decay Over Different Time Intervals

Example 1: An investment is initially worth $10,000. Write a formula for the value of this investment for each situation described

V(t) = investment value after t years
t = time in years

a) The value increases by 5% every year

t	V(t)

V(t) = ______________________

b) The value increases by 5% every 3 years

t	V(t)

V(t) = ______________________

c) The value increases by 5% every 6 months

t	V(t)

V(t) = ______________________

Example 2: In a recent disease epidemic, 1500 people were infected in one region of the United States. Using correct notation, write a function model for the number of people, P(t), infected in this region t years after 2000, assuming that the number of people infected:

a) Increases by 800 people every 2 years.

b**)** Increases by 20% every 3 years.

c) Decreases 10% every quarter

d) Decreases by 200 people every 6 months.

Example 3: A bacteria culture initially contains 1200 bacteria and doubles every half hour.

- Find the size of the bacteria population after 70 minutes
- Find the size of the bacteria population after 4 hours.

Example 4: The half-life of Palladium-100 is 4 days. After 14 days a sample of Palladium-100 has been reduced to a mass of 8 mg.

- What is the initial mass (in mg) of the sample?
- What is the mass 6 weeks after the start?

Section 11.10: Transformations and End Behavior

Example 1: Starting with the graph of f(x) = 2^x, write the equation of the graph that results from each of the following:

a) Shifts f(x) 3 units up.

b) Shifts f(x) 4 units right

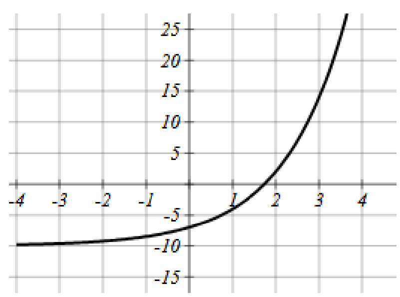
c) Reflects f(x) about the x-axis

d) Reflects f(x) about the y-axis

Example 2: Determine the end (long-run) behavior for each function

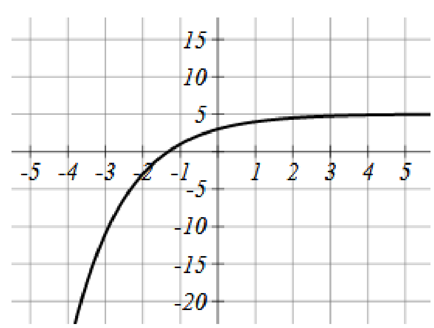
f(x) = $3(2)^x - 10$

- As x → ∞, f(x) → ____________
- As x → –∞, f(x) → ____________



f(x) = $-2(0.5)^x + 5$

- As x → ∞, f(x) → ____________
- As x → –∞, f(x) → ____________



Unit 12: Exponents and Logarithms

Section 12.1: Introduction to Logarithms

Properties of Exponents

$b^0 = 1$	$\frac{1}{b} = b^{-1}$	$\sqrt{b} = b^{1/2}$
$b^1 = b$	$\frac{1}{b^n} = b^{-n}$	$\sqrt[n]{b} = b^{1/n}$

Reading and Interpreting Logarithms

$$\log_b x = y$$

Read this as "Log, to the BASE b, of *x*, equals *y*"

This statement is true if and only if $b^y = x$

Meaning: Logarithms are exponents!

The logarithm (output of $\log_b x$) is the EXPONENT on the base, b, that will give you input *x*.

$$log_b(x) = a \quad \leftrightarrow \quad b^a = x$$

Properties of Logarithms

$\log_b x = y$	because	$b^y = x$
$\log_b 1 = 0$	because	$b^0 = 1$
$\log_b b = 1$	because	$b^1 = b$
$\log_b b^n = n$	because	$b^n = b^n$
$\log_b 0$ does not exist	because	There is no power of *b* that will give a result of 0.

Special Cases:

Common Logarithm: $\log(x) = \log_{10}(x)$

Natural Logarithm: $\ln(x) = \log_e(x)$

Example 1: Write the Logarithmic equations as Exponential equations:

$$log_b(n) = a \quad \leftrightarrow \quad b^a = n$$

$\log_x(z) = y$

$\log(n) = p$

$\ln(m) = p$

Example 2: Write the Exponential equations as Logarithmic equations:

$$log_b(n) = a \quad \leftrightarrow \quad b^a = n$$

$4^2 = 16$

$2^5 = 32$

$27^{1/3} = 3$

$7^{-2} = \frac{1}{49}$

Example 3: Compute each of the following logarithms without a calculator

a) $\log_2 2^4 =$	because	
b) $\log_2 4 =$	because	
c) $\log_3 27 =$	because	
d) $\log_8 1 =$	because	
e) $\log_5 \sqrt{5} =$	because	
f) $\log_4 4 =$	because	

Section 12.2: The Change of Base Formula

Example 1: Use a calculator to determine each value. Then write an exponential equation using the value.

1. ln(37)

2. log(295)

Change of Base Formula: $\boldsymbol{log_b x = \frac{log(x)}{log(b)} = \frac{ln(x)}{ln(e)}}$

Example 2: Use a calculator and the Change of Base Formula to determine each value. Then write an exponential equation using the value.

1. $\log_5(125)$

2. $\log_7(89)$

3. $\log_2(36)$

Section 12.3: Properties of Logarithms

Properties of Logarithms

- $\log_b(1) = 0$
- $\log_b(b) = 1$
- $\log_b(b^n) = n$
- $\log_b(x^n) = n\log_b(x)$
- $\log_b(xy) = \log_b(x) + \log_b(y)$
- $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$

Example 1: Combine into a single logarithm.

$2\log_5(x) + 3\log_5(2)$

$2\log(8) - 3\log(2)$

Section 12.4: Solving Logarithmic Equations

Solving logarithmic equations involves these steps:

1. **ISOLATE** the logarithmic part of the equation
2. Change the equation to **EXPONENTIAL** form
3. **ISOLATE** the variable
4. **CHECK** your result if possible
5. **IDENTIFY** the final result in **EXACT** form then in rounded form as indicated by the problem

Notes:

- To ISOLATE means to manipulate the equation using addition, subtraction, multiplication, and division so that the Log part and its input expression are by themselves.
- EXACT FORM for an answer means an answer that is not rounded until the last step

Example 1: Solve each logarithmic equations.

$\log_4(2x - 4) = 2$

$\log_2(x^2 - 2x) = 3$

Example 2: Solve each logarithmic equations. If needed, round to 4 decimal places.

$\log(x + 5) = \log(x) + \log(5)$

$\ln(x) + \ln(x - 4) = \ln(3x)$

Section 12.5: Solving Exponential Equations Using Logarithms

Solving exponential equations involves these steps:

1. **ISOLATE** the exponential part of the equation
2. **Take the log (or natural log) of both sides** of the equation, (NOTE: the equation may also be solved by converting the exponential equation to logarithmic form)
3. Use the property $\log(x^n) = n\log(x)$ to "bring down the exponent"
4. Isolate the variable

Example 1: Solve the exponential equations.

$5^x = 78$

$10^{-2x} = \frac{2}{3}$

Example 2: Solve the exponential equation. Give the exact solution and the solution rounded to 4 decimal places.

$$17 - 100(1.39)^t = -3083$$

Example 3: Solve for x by using like bases. (NOTE: If $a^m = a^n$, then $m = n$.)

$2^{x-3} = 2^{5x+11}$

$\left(\frac{1}{3}\right)^{x+2} = 3^{4x+6}$

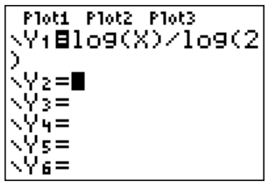
Section 12.6: Characteristics of Logarithmic Functions

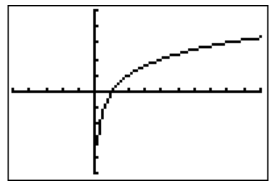
The Change of Base Formula can be used to graph Logarithmic Functions.

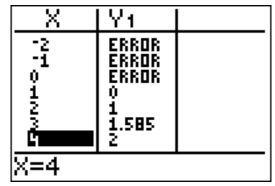
Worked Example: Given the function $f(x) = \log_2 x$, graph the function using your calculator and identify the characteristics listed below. Use window x: [-5..10] and y: [-5..5].

Graphed function: To enter the function into the calculator, we need to rewrite it using the Change of Base Formula, enter that equation into Y_1, and then Graph.

$$f(x) = \log_2 x = \frac{\log x}{\log 2}$$



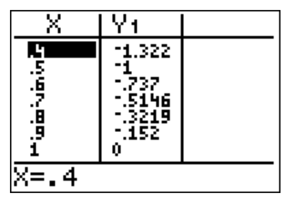




Characteristics of the Logarithmic Functions:

Domain: $x > 0$, Interval Notation: $(0,\infty)$
The graph comes close to, but never crosses the vertical axis. Any input value that is less than or equal to 0 ($x \le 0$) produces an error. Any input value greater than 0 is valid. The table above shows a snapshot of the table from the calculator to help illustrate this point.

Range: All Real Numbers, Interval Notation $(-\infty,\infty)$
The graph has output values from negative infinity to infinity. As the input values get closer and closer to zero, the output values continue to decrease (See the table to the right). As input values get larger, the output values continue to increase. It slows, but it never stops increasing.



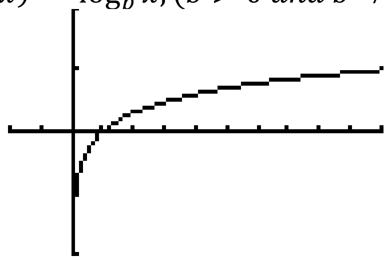
Vertical Asymptote at $x = 0$. The graph comes close to, but never crosses the line $x = 0$ (the vertical axis). Recall that, for any base b, $\log_b(0)$ does not exist because there is no power of b that will give a result of 0.

Vertical Intercept: Does Not Exist (DNE).

Horizontal Intercept: (1,0) This can be checked by looking at both the graph and the table above as well as by evaluating $f(1) = \log_2(1) = 0$. Recall that, for any base b, $\log_b(1) = 0$ because $b^0 = 1$.

Worked Example: All Logarithmic Functions of the form $f(x) = \log_b x$, ($b > 0$ and $b \neq 1$) share key characteristics. In this example, we look at a typical graph of this type of function and list the key characteristics in the table below.

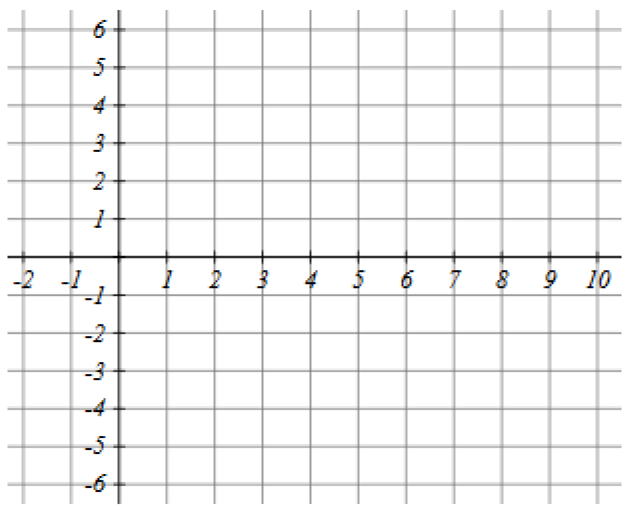
$$f(x) = \log_b x, (b > 0 \; and \; b \neq 1)$$



Domain	$x > 0$ (all positive real numbers) Interval Notation: $(0, \infty)$
Range	All real numbers Interval Notation: $(-\infty, \infty)$
Horizontal Intercept	(1, 0)
Vertical Asymptote	$x = 0$
Vertical Intercept	Does not exist
Left to Right Behavior	The function is always increasing but more and more slowly (at a decreasing rate)
Values of x for which $f(x) > 0$	$x > 1$ Interval Notation: $(1, \infty)$
Values of x for which $f(x) < 0$	$0 < x < 1$ Interval Notation: $(0, 1)$
Values of x for which $f(x) = 0$	$x = 1$
Values of x for which $f(x) = 1$	$x = b$ because $\log_b b = 1$

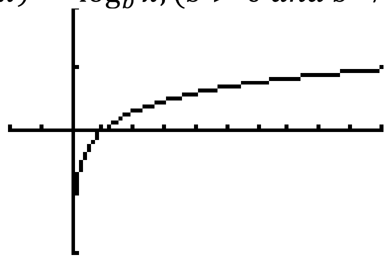
Example 1: Graph $y = \log_4(x)$ by hand. Give the domain, range, and the equation of the vertical asymptote.

x	y



Section 12.7: Finding the Domain and Asymptote of a Log Function

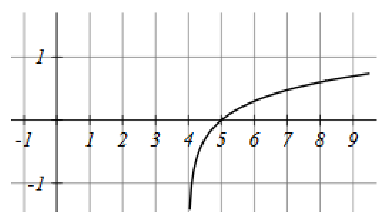
$$f(x) = \log_b x, (b > 0 \text{ and } b \neq 1)$$



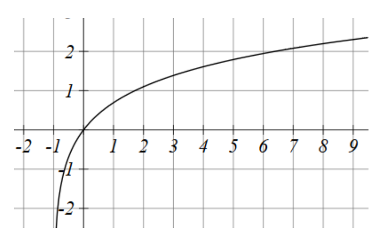
Domain	$x > 0$ (all positive real numbers) Interval Notation: $(0, \infty)$
Vertical Asymptote	$x = 0$

Example 1: Find the equation of the vertical asymptote to each function.

f(x) = log(x – 4)



f(x) = ln(x + 1)



Example 2: Find the domain of the following functions.

$y = \log_2(2x - 4)$ $f(x) = \log(x^2 - 3x)$

Section 12.8: Inverses

Logarithmic Functions and Exponential Functions are INVERSES of each other.

(Recall) Properties of Inverse Functions:

- **Input and Output**: The input of f is the output of f^{-1} and vice versa
- **Domain and Range**: The domain of f is the range of f^{-1} and vice versa.
- **Composition**: $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.
- **Symmetry**: The graphs of f and f^{-1} are symmetric about the line y = x.

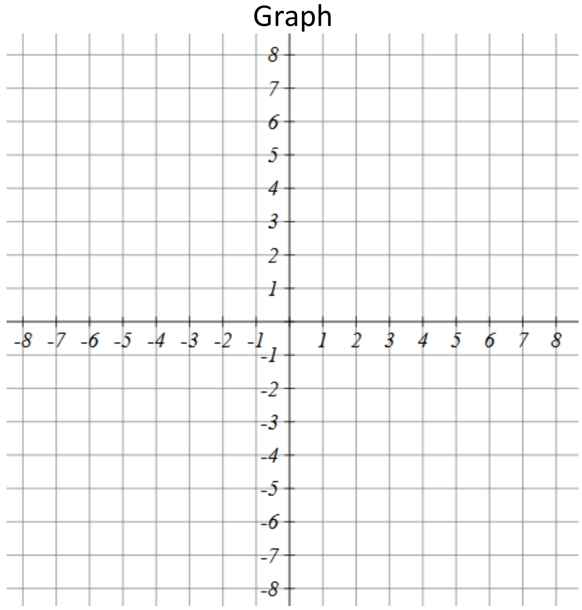
Characteristic	$f(x) = b^x$	$g(x) = log_b(x)$
Domain	$(-\infty,\infty)$	$(0,\infty)$
Range	$(0,\infty)$	$(-\infty,\infty)$
Horizontal Intercept	DNE	(1, 0)
Vertical Intercept	(0,1)	DNE
Asymptote	y = 0 (the x-axis)	x = 0 (the y-axis)
Composition	$f(g(x)) = b^{log_b(x)} = x$	$g(f(x)) = log_b(b^x) = x$

Example 1: Graph the functions $y = 2^x$ and $y = log_2(x)$ on the same coordinate plane.

$y = 2^x$

x	y
0	
1	
2	
3	
-1	
-2	

Graph



$y = log_2(x)$

x	y
	0
	1
	2
	3
	-1
	-2

Section 12.9: Convert ae^{kt} to ab^t

Example 1: Given $f(t) = 356e^{0.19t}$. What is the continuous rate? What is the annual rate?

Example 2: Given $f(t) = 356e^{-0.04t}$. What is the continuous rate? What is the annual rate?

Section 12.10: Using Logarithms to Solve Exponential Equations (Applications)

Example 1: The value of China's exports of automobiles and parts in billions of dollars is approximately $V(t) = 1.8415e^{0.3389t}$, where t=0 corresponds to 1998. In what year will the export value reach $8.2 billion?

Example 2: A wooden artifact from an ancient tomb contains 60% of the carbon-14 that is present in living trees. How long ago, to the nearest year, was the artifact made? The half-life of carbon-14 is 5730 years.

NOTE: Another way to find the solution to Example 2 is to use the formula $f(t) = ab^t$. Assume again that the initial amount was 1g, so is now 0.6g (because only 60% remains). Since we know that the half-life is 5730 years (meaning that the amount of carbon-14 decays by half every 5730 years), we can write this equation as: $1\left(\frac{1}{2}\right)^{t/5730} = 0.6$, or just $\left(\frac{1}{2}\right)^{t/5730} = 0.6$.

To solve this, take the log (or natural log) of both sides: $log\left(\frac{1}{2}\right)^{t/5730} = log(0.6)$

Bring down the exponent: $\frac{t}{5730} log\left(\frac{1}{2}\right) = log(0.6)$

Isolate t: $t = 5730\,\frac{log(0.6)}{log\left(\frac{1}{2}\right)} \approx 4223\ years$

Example 3: At the beginning of an experiment, a scientist has 200g of radioactive goo. After 40 minutes, her sample has decayed to 15.1g.

- Find a formula for the amount of goo remaining at time t.

- What is the half-life of the goo in minutes?

- How many grams of goo will remain after 71 minutes?

Unit 13: Systems and Matrices

Section 13.1: General Form for a Linear Equation: *ax + by = c*

Slope-Intercept Form of a Linear Equation	**General (Standard) Form of a Linear Equation**
$y = mx + b$ x = input, y = output m = slope b = vertical intercept (0, b)	$ax + by = c$ x = input, y = output a, b, and c are constants

Example 1: Consider the linear equation $3x - 5y = 30$

a. Write this equation in slope-intercept form.

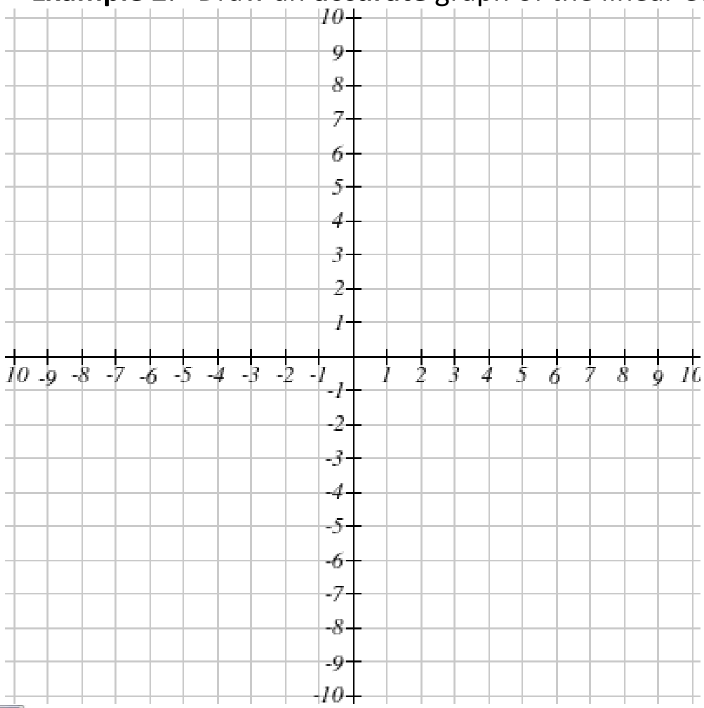
b. Identify the slope.

Determining Intercepts:
To find the **vertical intercept**, set $x = 0$ and solve for y.
To find the **horizontal intercept**, set $y = 0$ and solve for x.

c. Determine the vertical intercept.

d. Determine the horizontal intercept.

Example 2: Draw an **accurate** graph of the linear equation $3x + 2y = 16$.



Slope-Intercept Form:______________

Slope: ________

Vertical Intercept: ________

Horizontal Intercept: ________

Additional points on the line:

____

Example 3: Movie tickets cost $7 for adults (matinee), $5.50 for children. A total of $668 was collected in ticket sales for the Saturday matinee.

a. Write an equation representing the total amount of money collected.

b. If 42 adult tickets were purchased for this matinee, how many children were there?

Example 4: Juan has a pocket full of dimes and quarters. The total value of his change is $6.25.

a. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.

b. If Juan has 7 quarters in his pocket, how many dimes are there?

Section 13.2: Systems of Linear Equations

Definitions
Two linear equations that relate the same two variables are called a **system of linear equations.**
A **solution** to a system of linear equations is an **ordered pair** that satisfies both equations.

Example 1: Verify that the point (5, 4) is a solution to the system of equations

$$y = 2x - 6$$
$$y = x - 1$$

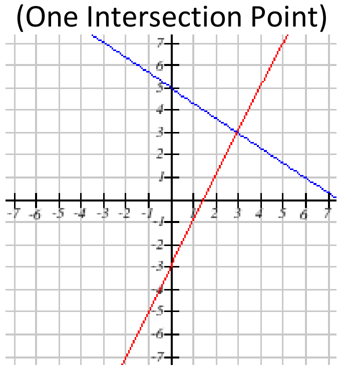
Types of Solutions to a Linear System of Equations

Graphically, the solution to a system of linear equations it a point at which the graphs intersect.

Types of Solutions to a Linear System of Equations:

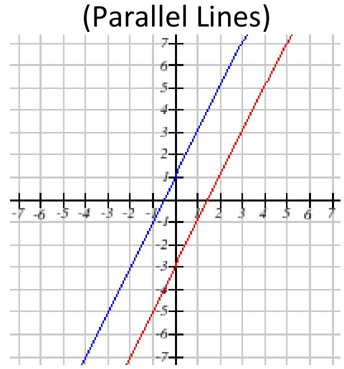
- **One unique solution**: The lines intersect at exactly one point
- **No solution**: The two lines are parallel and will never intersect
- **Infinitely many solutions**: This occurs when both lines graph as the same line

One Unique Solution
(One Intersection Point)



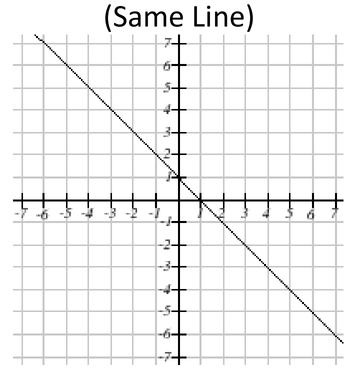
Consistent and Independent

No Solution
(Parallel Lines)



Inconsistent

Infinitely Many Solutions
(Same Line)

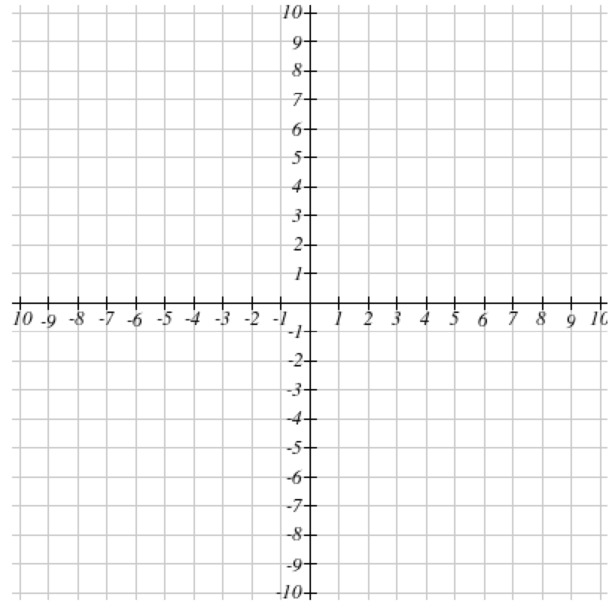


Consistent and Dependent

Solving a System of Linear Equations by Graphing

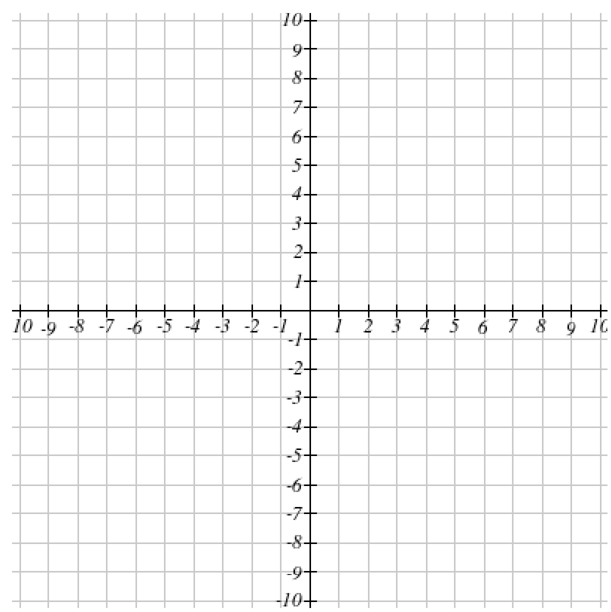
Example 2: Solve the system of equations by graphing. Check your answer.

$$y = 6 - \frac{2}{3}x$$
$$y = x + 1$$



Example 3: Solve the system of equations by graphing. Check your answer.

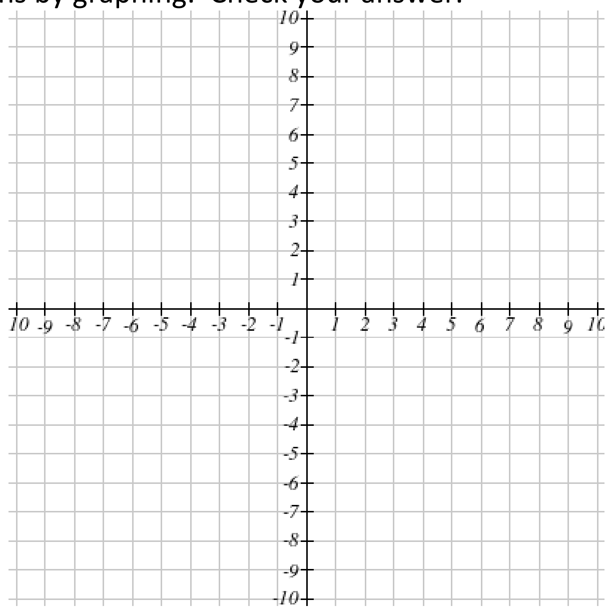
$$4x - 3y = -18$$
$$2x + y = -4$$



Example 4: Solve the system of equations by graphing. Check your answer.

$x - 3y = 3$

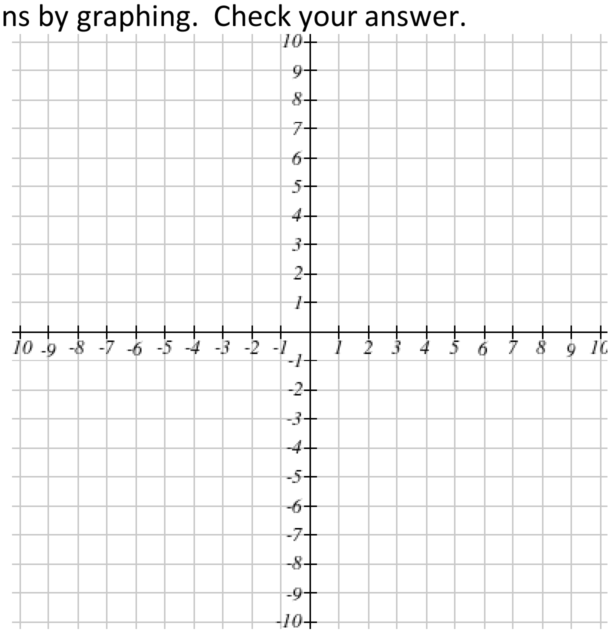
$3x - 9y = -18$



Example 5: Solve the system of equations by graphing. Check your answer.

$2x + y = 3$

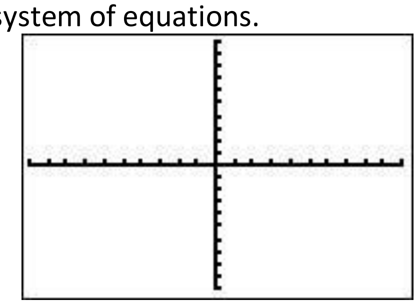
$6x + 3y = 9$



Example 6: Use your graphing calculator to solve the system of equations.

$3x + 2y = 6$

$x - 2y = -6$



Section 13.3: The Substitution Method

Consider the following equations: $y = 2x$
$x + y = 3$

Using Substitution to Solve a Linear System of Equations
Step 1: Solve one of the equations of the system for one of the variables.
Step 2: Substitute the expression for the variable obtained in step 1 into the other equation.
Step 3: Solve the equation.
Step 4: Substitute the result back into one of the original equations to find the ordered pair solution.
Step 5: Check your result by substituting your result into either one of the original equations.

Example 1: Solve the system of equations using the Substitution Method.

$3x - 2y = 16$
$2x + y = 20$

Example 2: Solve the system of equations using the Substitution Method.

$5x - 4y = 9$
$x - 2y = -3$

Example 3: Solve the system of equations using the Substitution Method.

$3x + y = 5$

$6x + 2y = 11$

Example 4: Solve the system of equations using the Substitution Method.

$x - y = -1$

$y = x + 1$

Section 13.4: The Addition (Elimination) Method

Consider the following systems of equations:

$x - 2y = -11$
$5x + 2y = 5$

Using the Addition (Elimination) Method to Solve a Linear System of Equations
Step 1: "Line up" the variables.
Step 2: Determine which variable you want to eliminate. Make those coefficients opposites.
Step 3: Add straight down (one variable should "drop out")
Step 4: Solve resulting equation
Step 5: Substitute this result into either of the ORIGINAL equations
Step 6: Solve for the variable
Step 7: CHECK!!!!!!! Plug solution into BOTH equations!

Example 1: Solve the system of equations using the Addition (Elimination) Method.

$4x - 3y = -15$
$x + 5y = 2$

Example 2: Solve the system of equations using the Addition (Elimination) Method.

$3x - 2y = -12$
$5x - 8y = 8$

Example 3: Solve the system of equations using the Addition (Elimination) Method.

$7x - 2y = 41$
$3x - 5y = 1$

Section 13.5: Applications

Example 1: Movie tickets cost $7 for adults (matinee), $5.50 for children. There are 218 seats in the theater. A total of $1,463 was collected in ticket sales for the sold-out Saturday matinee. How many adults and how many children were in the theater?

Example 2: Jun had $26,100 and chose to split the money into two different mutual funds. During the first year, Fund A earned 7% interest and Fund B earned 2% interest. If Jun received a total of $1,107 in interest, how much had he invested into each account?

Example 3: The Nut Shack sells hazelnuts for $6.80 per pound and peanuts for $5.30 per pound. How much of each type should be used to make a 37 pound mixture that sells for $5.91 per pound? Round your answers to the nearest pound.

________pounds of hazelnuts

________pounds of peanuts

Example 4: A chemist needs to make 2 liters of a 15% acid solution from a 10% acid solution and a 35% acid solution. How many liters of each solution should she mix to get the desired solution?

Section 13.6: Introduction to Matrices

A **matrix** is a rectangular array of numbers. A **row** in a matrix is a set of numbers that are aligned horizontally. A **column** in a matrix is a set of numbers that are aligned vertically. Three examples are shown below.

$$A = \begin{bmatrix} 1 & -8 \\ 6 & 11 \end{bmatrix} \qquad B = \begin{bmatrix} 9 & 2 & 7 \\ 0 & -3 & 6 \\ -5 & 8 & 12 \end{bmatrix} \qquad C = \begin{bmatrix} 4 & 2 \\ 0 & 1 \\ -9 & -7 \end{bmatrix}$$

We often describe a matrix by its dimensions: m x n indicating m rows and n columns. For example, a "2 x 5 matrix" would have 2 rows and 5 columns. The matrix A above is a 2 x 2 matrix. Matrix B is a 3 x 3 matrix. Matrix C is a 3 x 2 matrix.

A matrix can serve as a tool for representing and solving a system of equations. To express a system in matrix form, the coefficients and constants become the entries of the matrix. A vertical line is sometimes drawn to separate the coefficients from the constants (replacing the equals signs). When a system is written in this form, we call it an **augmented matrix**.

System of Equations	Augmented Matrix
3x – 5y = 9 2x + 6y = 1	$\left[\begin{array}{cc\|c} 3 & -5 & 9 \\ 2 & 6 & 1 \end{array}\right]$

Example 1: Write the system of equations as an augmented matrix.

-5r + z + 2n = 300
z = 100
2r – n = 150

Example 2: Solve a system of equations using your graphing calculator. Use your graphing calculator to verify that you answer is correct.

2x – 4y + z = 13
-x + y – z = -4
x – 2y = 7

Section 13.7: Applications of 3x3 Systems of Equations

Example 1: Bill is trying to plan a meal to meet specific nutritional goals. He wants to prepare a meal containing rice, tofu, and peanuts that will provide 246 grams of carbohydrates, 92 grams of fat, and 62 grams of protein. He knows that each cup of rice provides 40g of carbohydrates, 0 g of fat, and 1g of protein. Each cup of tofu provides 7g of carbohydrates, 10g of fat, and 15g of protein. Finally, each cup of peanuts provides 32g of carbohydrates, 72g of fat, and 27g of protein. How many cups of rice, tofu, and peanuts should he eat?

Example 2: Maricopa's Success Scholarship Fund receives a gift of $200,000.
The money is invested in stocks, bonds, and CD's.

CD's pay 3.1% interest,
bonds pay 4.25% interest,
and stocks pay 8.2% in dividends.

Maricopa success invests $25,000 more in bonds than in stocks.

If the annual income from the investments is $9,925, how much was invested in each account?